Terrain visibility-dependent facility location through fast dynamic step-distance viewshed estimation within a raster environment

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Abstract

The placement of facilities, such as radar, telecommunication towers, telescopes, surveillance cameras and watchtowers, requires careful planning. To operate optimally, these facilities have to be placed according to very specific terrain-related requirements which vary widely, such as intervisibility, proximity and sunlight exposure. Another important criterion to consider for facility placement is that of the viewshed of the facility, which is a representation of the visible area of terrain surface within a specified perimeter around the facility. From this viewshed a viewshed visibility percentage (VVP) may be computed, indicating the percentage of terrain surface lying within the analysed viewshed area that is visible to the facility. The focus in this paper falls on the estimation of such VVPs — specifically aiming to reduce the computation time of viewsheds through estimated results instead of exact ones, while simultaneously aiming to minimise loss of accuracy. Firstly, the notion of a viewshed and the computation thereof in a raster data environment is elucidated upon, after which a method for estimating viewsheds is proposed. The estimation method relies on resolution-sensitive analyses performed at angular intervals from the facility location. A case study involving a large area of terrain is then performed and the results and effectiveness of the proposed methods are investigated within the context of a bi-objective optimisation model.

Key words: Terrain modelling, line-of-sight, viewshed analysis, facility location

1 Introduction

The class of facility location problems is well documented in the operations research literature [1, 2, 6]. The prototype example of a facility location problem in the manufacturing sector is the optimal placement of a number of factories with the objective of minimising combined transportation costs and delivery times between these factories. It is, however, possible to adapt traditional facility location problem formulations to include placement criteria involving the physical terrain surrounding the facilities. An example of a single-objective facility location problem involving (inter)visibility and terrain-related facility location criteria is the problem of locating \( n \) radar facilities on the earth’s surface with the objective of maximising the proportion of some pre-specified area of the terrain surface that is visible to at least \( m \) of the radar facilities, with \( m < n \). The computational complexity of solving this problem grows substantially as a function of increasing values of the parameters \( m \) and/or \( n \), as a function of increasing resolution of terrain representation data and as a function of increasing range of the pre-specified area of the terrain surface over which the problem is solved. When the problem becomes multi-objective, multiple terrain-related objectives and additional constraints magnify the increase in model complexity (very substantially). The reason for this magnification lies in the conflicting nature of the objectives. For example, increasing a

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telecommunication tower’s terrain surface visibility may result in a decrease in the number of towers visible from that tower which can see the fewest remaining towers because of terrain interference.

In order to arrive at good candidate solutions to facility location problems, such as those mentioned above, powerful single- or multi-objective optimisation metaheuristic procedures (such as simulated annealing or genetic algorithms) are typically required\(^1\). However, since it may be very costly (in terms of the number of floating point operations performed) to evaluate the desirability of a candidate solution in terms of the model objective functions, extreme care should be taken to employ temporally efficient yet sufficiently accurate terrain-related analyses. The purpose of this paper is to make a recommendation in this respect by considering terrain data resolution effects on viewshed analyses.

The criterion of terrain surface visibility refers to the portion of terrain visible from a specific location, which is limited by terrain interference. Here visibility may refer either to optical visibility or merely to telecommunications or radar detection capability. The determination of viewshed visibility percentages (VVPs) depends heavily on factors such as data resolution, terrain roughness and distance scale.

This paper opens in §2 with an explanation of the notion of viewshed analysis and the computations involved, specifically in a raster (gridded) terrain environment. An investigation into previously analysed data that exhibit interesting characteristics in terms of viewshed analysis follows in §3, leading to the proposal of a viewshed estimation method which aims to take advantage of certain terrain analysis characteristics in order to compute (as estimations) VVPs with the aim of finding an acceptable trade-off between maximising computation speed and minimising loss of accuracy, in §4. An estimation of viewsheds is thereafter formulated as a bi-objective optimisation problem in §5. Section 6 is a case study in which the newly proposed viewshed estimation technique is applied. Some ideas with respect to possible future work are presented in the final section.

2 Viewshed analysis in a raster environment

Typically, the determination of viewsheds may be performed with respect to two data structure types employed to represent the earth’s terrain surface, viz. (1) raster data, or (2) Triangulated Irregular Networks (TINs). Raster data, otherwise known as digital elevation models, represent the earth’s surface as a matrix of elevations of regularly spaced points (also called gridposts) above sea-level. The gridposts in raster data are measured uniformly across latitude and longitude and may additionally be manipulated to achieve uniform spacing (which is standard practice [3]). TINs are derived from raster data, identifying important gridposts from the raster data model with respect to terrain relief characteristics, while simultaneously disregarding other, lesser important gridposts. The important gridposts are then joined to each other by straight lines to create planar triangles which represent the terrain surface. The effectiveness of using raster data and TINs have been compared extensively before (see [4, 7], for example). The results of these comparisons indicate that no clear preferred data structure type is evident for viewshed determination. For the sake of simplicity, raster data are therefore used in the determination of viewsheds in this paper.

Figure 1 contains an example of what a viewshed may look like for a central gridpost, called the observer, within a 50 kilometre radius around it. The darker shaded areas within the circle are portions of the earth’s surface that are invisible to the observer. Figure 2 contains an explanation of the process involved in computing a viewshed within a raster environment. In Figure 2 (a), viewed from above, a perimeter is specified within which the viewshed of the central observer gridpost is required. From the observer gridpost, a number of angular analyses are performed. Moving away from the gridpost along each azimuth in fixed step distances, gridposts are identified which lie nearest the azimuth line after each step, after which these target gridposts are determined to be either visible or invisible to the observer. Three such azimuths and their resulting target gridposts are shown in the figure. For a complete analysis, angles within the range \([0, 360)\) have to be considered. It may, however, be specified that only a specific sector or a combination of sectors are to be considered in the event that only certain important terrain surface areas are of concern. Whichever the specification, the spacing between the azimuths is a crucial factor of the analysis. It is clear in Figure 2 (a), where the azimuth spacings are very large, that an excessive number of gridposts are excluded from the analysis if the spacings are too large. This is an important

\(^1\)See [5] for an example of a single-objective facility location problem featuring multiple observer and combined terrain visibility objectives using such metaheuristic procedures.
Figure 1: An example of viewshed analysis with respect to an observer gridpost (the black dot) on the earth’s surface within a specified radius of 50 kilometres. The darker shaded areas represent earth surface portions that are invisible to the observer as a result of terrain obstruction.

fact to consider in cases where a complete terrain surface analysis is desired.

Figure 2 (b) contains an example of an analysis along a single azimuth. In this example, ten target gridposts are analysed, of which six are determined to be visible to the observer. This results in a gridpost visibility percentage of 60% along the particular azimuth which, in general, is used as indication of overall terrain surface visibility along the azimuth. Suppose that $\mathcal{A}$ denotes the set of angles analysed around the observer gridpost and denote the number of angles by $n_a = |\mathcal{A}|$. Also, let $v(\theta, r)$ denote the visibility percentage along azimuth $\theta \in \mathcal{A}$ within a specified range $r$ from the observer. If $n_v(\theta, r)$ denotes the number of visible gridposts along the azimuth and $n_t(\theta, r)$ denotes the total number of target gridposts analysed, then

$$v(\theta, r) = \frac{n_v(\theta, r)}{n_t(\theta, r)} \times 100 \%.$$  \hspace{1cm} (1)

The VVP is defined as the mean visibility percentage determined over all the angles in $\mathcal{A}$. The VVP may therefore be computed as

$$V(r) = \frac{\sum_{\theta \in \mathcal{A}} v(\theta, r)}{n_a} \%.$$  \hspace{1cm} (2)
3 Analysis of viewshed patterns under different data resolutions

The work presented in this paper is the result of numerous previous viewshed analyses, of which the results consistently exhibited important similarities and characteristics at varying raster data resolutions. One such analysis is discussed to illustrate these similarities and characteristics.

The use of Digital Terrain Elevation Data (DTED) is standard practice when modelling and simulating terrain [3]. A single DTED file contains earth surface elevation data for a one degree by one degree area of latitude and longitude. These data files are available at various resolutions and comprise square digital arrays of elevation data. The standard resolutions of DTED files generally have spacings between gridposts of approximately 1 kilometre for DTED level 0 (DL0), 100 metres for DTED level 1 (DL1) and 30 metres for DTED level 2 (DL2), respectively, though varying slightly as a function of latitude North or South from the equator. The categorisation of raster data into different levels of resolution files may be seen as superfluous, since the data in a DL0 file is simply a subset of the data in a DL1 file, and likewise for DL1 with respect to DL2 files. It is therefore not necessary to store different levels of data files — storing the highest resolution data available and extracting gridposts if lower resolution data are required is a more efficient way of storing data, while at the same time facilitating the possibility of extracting data at different resolutions than the standard DTED ones.

The results of a viewshed analysis performed at the three DTED resolutions from the same observer gridpost are shown in Figure 3. Three important sectors are identified and indicated in the figure. Consider sector 1. The visibility of terrain at DL0 is remarkably higher than those at higher resolutions. This is a result of the resolution of DL0 data with respect to points near the observer gridpost. Because the distances between the gridposts are so large, important features of the terrain are skipped, resulting in the terrain interference at low resolution being reduced close to the observer point, resulting in increased visibility of far lying target gridposts. For this specific analysis and observer it is interesting to note that there are high-altitude points very close to the observer, which are skipped in the low resolution analyses, but which increasingly affect visibility as they are included at higher resolutions. At the highest resolution there is no visibility of terrain surface in sector 1, due to gridpost interference adjacent to the observer gridpost. It follows that the use of the highest resolution data available in the vicinity of the observer gridpost is essential to the accuracy of a viewshed analysis further away from the observer.

Considering sectors 2 and 3, it is interesting that the patterns of visible target gridposts further away from the observer are very similar at all three resolution levels, despite some interference near the observer gridpost at higher resolutions. This would result in similar VVPs for the sectors in question beyond the ranges of interference if the observer gridpost were not to experience the interference it does in its close vicinity. This indicates that, at least for viewshed analyses, using the maximum resolution data available becomes less important further away from the observer gridpost.

Observations similar to those above were consistently made in other viewshed analyses, resulting in the following conclusions with respect to viewshed analysis accuracy:

- The highest resolution data available should always be used for viewshed analysis in the vicinity of the observer gridpost.
- The importance of high-resolution data required for viewshed analysis decreases as the distance from the observer gridpost increases.

4 Viewshed estimation through dynamic step distance

Following the conclusions made in §3, a dynamic step distance method is proposed in this section for the computation of angular terrain visibility percentages — with the aim of facilitating the estimation of VVPs at reduced computation times.

As discussed in §2 and illustrated in Figure 2, the computation of VVPs requires analyses along a set of azimuths at fixed step distances and determining the nearest target gridposts to be evaluated in terms of their visibility with respect to the observer gridpost along each azimuth in question. The computation time of such an analysis depends heavily on the number of target gridposts analysed which, in turn, depends on the step distance used. If the number of gridposts analysed per azimuth is decreased by
increasing the step distance, the overall VVP computation time would decrease as a result. However, increasing the step distance is comparable to performing analyses at lower resolutions which are expected to return less accurate results than those obtained at higher resolutions — most notably as a result of terrain interference considerations in the vicinity of the observer gridpost, as discussed in §3.

It may be possible to adapt the step distance with the aim of maximising accuracy in the nearby vicinity of the observer gridpost, while minimising the total number of target gridposts by increasing the step distance as the distance from the observer gridposts increases. This incremental step-distance process is similar to the concept of compound interest on financial investments — a slow initial increase in investment amount followed by exponential increase, with the rate of increase depending on the interest rate. If the initial smallest step distance is seen as the initial investment amount, with the rate of change in the step distance seen as the interest rate of the investment, a formula for the factor by which the initial step distance is to be scaled at each step is given by the standard formula

$$f(n) = (1 + i)^n,$$

where $f(n)$ denotes the factor by which the initial step distance is scaled at the $n$th step (investment growth after $n$ years) and $i$ denotes the rate of increase in the factor (investment interest rate). Additionally, due to the terrain-specific nature of the data, it may be desirable to define an initial window range from the observer gridpost, $r_w$, within which the step distance remains at the smallest possible value (therefore a factor of 1), before switching over to the incremental adaptation in (3). This will ensure that the analysis in the vicinity of the observer gridpost remains at the highest level resolution possible within the window range, with the aim of improving the accuracy of estimated VVPs as a result of the importance of high resolution near the observer. Furthermore, a maximum step distance factor, $f_M$, should also be enforced to keep the step distance within reasonable levels, since analyses over large distances or larger values of $i$ may ultimately result in impractically large step distances due to the exponential rate of increase in the factor.

Figure 4 contains an example of the suggested profile of the step-distance scaling factor against the distance from the observer in terms of the parameters introduced above.

The values assigned to $r_w$, $i$ and $f_M$ have to be considered carefully. If $r_w$ is very small, the step distance
may start increasing too close to the observer, resulting in a large loss of accuracy. Moreover, large values of $i$ may result in the step distance growing too fast, while small values of $i$ may result in the step distance not growing fast enough to reduce the number of target gridposts analysed. The value of $f_M$ is equally important, because values of $f_M$ that are too large or too small may result in very large eventual step distances or minimal step distance increase gains, respectively. The above-mentioned parameters are, furthermore, sensitive to the overall range over which the viewshed analysis is to be performed. It is therefore necessary to determine suitable combinations of these parameters that yield acceptable results (both in terms of accuracy and speed) for the viewshed range in question.

5 Formulation of a bi-objective viewshed estimation model

The trade-off achieved between the accuracy of viewshed results and the speed-up in computation times required to arrive at these results depends on the choice of the initial window range $r_w$, the scaling factor growth rate $i$ and the maximum scaling factor $f_M$, described in the previous section. The two criteria — viewshed accuracy and decrease in computation time — may be modelled as objectives in a bi-objective optimisation viewshed estimation model.

The objectives of viewshed estimation are, of course, conflicting in nature — reducing the computation time of a viewshed typically results in a loss of accuracy. A trade-off between extremal values of the objective functions is therefore sought in the form of a subset of candidate solutions to the model (where a candidate solution is a combination of values for the parameters $r_w$, $i$ and $f_M$), known as a Pareto-optimal set of solutions or a non-dominated set of solutions. This set of solutions has the property of being superior to the remaining points in the solution space with respect to all the objectives, while at the same time being inferior to the solutions of one or more, but not all, of the objectives in the Pareto-optimal set.

Let the VVP of an observer gridpost within a specified range $r$, determined at the smallest possible uniform step distances (highest resolution), be given by the expression for $V(r)$ in (2). Then, let the estimated VVP of the observer within the same specified range using factor-scaled step distances be denoted by $V'(C, r)$, where $C$ is a parameter combination of values for $r_w$, $i$ and $f_M$. Then the VVP estimation percentage error, $V_e(C, r)$, is determined by

$$V_e(C, r) = \frac{V'(C, r) - V(r)}{V(r)} \times 100\%.$$  \hspace{1cm} (4)

Similarly, let $T(r)$ denote the time spent computing the viewshed of an observer gridpost at uniform minimum step distances and let $T'(C, r)$ denote the time spent computing the estimated viewshed under the parameter combination $C$. Then the percentage decrease in computation time, $T_d(C, r)$, is determined as

$$T_d(C, r) = \frac{T(r) - T'(C, r)}{T(r)} \times 100\%.$$  \hspace{1cm} (5)
To determine near optimal values for the parameter combination \( C \) with respect to different values of \( r \), the viewshed estimation model requires that multiple observer gridposts be analysed, after which mean VVP percentage errors and mean percentage decreases in computation time are calculated for different \( C \). These mean VVP percentage errors and mean percentage decreases in computation time may be used to determine which combinations of \( C \) return suitable trade-off results with respect to the objectives. Let \( \mathcal{O} \) be a population of observer gridposts analysed to determine mean percentage values of \( V_e(C,r) \) and \( T_d(C,r) \), where the number of observers is \( n_o = |\mathcal{O}| \). Also, let \( e_M \) denote the maximum mean VVP error percentage threshold values considered acceptable, while \( t_m \) denotes the minimum mean percentage decrease in computation time threshold value considered acceptable. The objectives of the bi-objective model is then to find combinations of \( C \) that

\[
\text{minimise} \quad V_e(C,r) = \frac{\sum_{o \in \mathcal{O}} V_e(C,r)}{n_o}, \\
\text{maximise} \quad T_d(C,r) = \frac{\sum_{o \in \mathcal{O}} T_d(C,r)}{n_o},
\]

subject to the constraints

\[
V_e(C,r) < e_M , \quad T_d(C,r) > t_m.
\]

6 Viewshed estimation case study

In §4 a dynamic step distance viewshed estimation method was proposed, with the aim of reducing the viewshed computation time at a minimal loss of accuracy. A case study utilising the dynamic step-distance method and the bi-objective viewshed estimation model of §5 to find near-optimal and practically feasible values for the parameter combination \( C \) is provided in this section.

In order to evaluate the objective functions in (4) and (5), a set of observer gridposts is required. For this purpose a 250 $\times$ 150 kilometre section of terrain was selected, shown in Figure 5. In this area, a point selection area was identified within which fifty sample observer gridposts were randomly selected, as indicated in the figure. To simulate real life scenarios with respect to facilities such as telecommunication towers and radars, the observer was placed at an offset of 10 metres above the terrain surface. Additionally, these types of facilities would typically be placed in areas with reasonably high visibility. Therefore, only observers with a minimum VVP of 15% were selected, where the viewshed range \( r \) was chosen to be 30 kilometres. The initial window range values of \( r_w \) were chosen at uniform intervals of 2 500 metres, starting at 0 and ending at 15 000 metres. The factor change rate \( i \) was chosen at uniform intervals of 0.05%, starting at 0.05% and ending at 0.5%. The maximum factor scale was simply chosen as \( f_M = 16 \). The azimuth angles analysed were at 1 degree intervals in the range \([0, 360)\). The resolution of terrain data was chosen as DL2 (the smallest step distances are therefore approximately 30 metres).

The results of the analysis are presented in Figure 6 for all seventy possible choices of the combination \( C \). The pareto front of non-dominated solutions is shown as filled markers (there are twenty four non-dominated solutions) and the relevant values corresponding to the pareto front are summarised in Table 1 and listed in order of increasing mean decrease in operation time. The standard deviations of the mean error percentages are also provided. Analysing the results of the pareto optimal solutions provides encouraging results, indicating the potential benefit of the dynamic step-distance method in estimating VVPs. A particularly good result is that of the combination of \( r_w = 0 \) metres and \( i = 0.05\% \), with a mean percentage decrease in computation time of 20.1% while suffering only a 10% mean percentage error. The results may be refined further by enforcing the constraints of the form (8) and/or (9).

7 Future work

The work described in this paper is ongoing research. The results presented here are based on the results of a number of viewshed analyses. In order to further validate and improve the dynamic step-distance
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Table 1: Solution data for the solutions along the pareto front in Figure 6, numbered the same as in the figure.
Figure 5: A $250 \times 150$ kilometre section of terrain chosen to investigate the potential benefit of the dynamic step-distance method of §4 in the context of the bi-objective viewshed estimation model of §5. The fifty sample observer gridpoints used in the study are indicated by dark markers.

Figure 6: Mean VVP percentage errors and mean percentage decreases in computation time for the different combinations of values for the parameters $r_w$ and $i$, at a viewshed range of $r = 30$ kilometres and maximum scaling factor of $f_M = 16$.

method, such as determining suitable values for $r_w$, $i$ and $f_M$ at different viewshed ranges, more extensive analyses should, however, be performed on various terrain areas. The preliminary results obtained here nevertheless indicate that optimisation possibilities exist and that the method proposed in this paper warrants further research. Such further research may include analysing the effect of different values of $f_M$ and $r$ on the results of viewshed analyses using the dynamic step-distance method. The concentration of target gridpoints of estimated viewsheds vary at different combinations of $C$ as a function of distance from the observer. Estimated viewsheds are therefore expected to have VVPs closer to that of the maximum resolution analyses if a distance-weighted gridpost approach is followed.

References


