Weapon assignment decision support in a surface-based air defence environment

DP Lötter* & JH van Vuuren†

November 19, 2013

Abstract

A surface-based air defence environment contains defended assets on the surface of a land area, lake or ocean which require protection from aerial threats entering the defended airspace. Protection is typically afforded by means of pre-deployed surface-based weapon systems. These weapon systems are assigned to engage aerial threats according to a pre-specified criterion or set of criteria, a problem known as the weapon assignment problem. A human operator is responsible for solving this problem under severely stressful conditions. A computerised threat evaluation and weapon assignment decision support system may, however, be employed to provide real-time decision support to the operator. Such a decision support system typically consists of two subsystems known as the threat evaluation subsystem and the weapon assignment subsystem.

The aim of this paper is to provide a generic design framework for a weapon assignment subsystem. This includes a discussion on the various substructures collected within a weapon assignment subsystem, as well as the components within each of these substructures. In particular, four classes of weapon assignment models, each having different characteristics, are proposed for inclusion in the weapon assignment subsystem. Examples of models in these classes are solved in the context of a simulated, but realistic surface-based defence scenario to illustrate the working of the proposed weapon assignment subsystem.

Keywords: Weapon assignment, multi-objective, decision support, surface-based air defence.

1 Introduction

On 3 July 1988, a civilian jet airliner carrying 290 passengers and crew members was flying over the Persian Gulf en route to Dubai in the United Arab Emirates. The United States Navy guided missile cruiser, the USS Vinceness (CG 49), operating in the Persian Gulf at that time, mistakenly identified the airliner as a hostile F-14 Tomcat fighter aircraft. After the airliner failed to respond to multiple communication attempts from the USS Vinceness, it fired two radar-guided missiles at the airliner “in self-defence.” The consequences of this decision were catastrophic, resulting in the deaths of all 290 passengers and crew members on board the airliner. The disaster prompted a number of formal investigations by the United States Navy and the International Civil Aviation Organisation (McCarthy, 1991) which concluded that the misidentification of the aircraft may have been due to the compression of time in conjunction with the contemporaneous engagement with Iranian gunboats as well as a psychological phenomenon known as scenario fulfilment³.

As a result of the incident mentioned above, the United States Office of Naval Research sponsored research and the development of a program called Tactical Decision Making Under Stress (TADMUS). The main
focus of this program was to improve the combat performance of team operators under stress by means of enhanced training and to provide them with the necessary Decision Support Systems (DSSs) (Cannon-Bowers, Johnston and Sales, 1998; Johnston and Paris, 1999). The aim of such a DSS, commonly known as a Threat Evaluation and Weapon Assignment DSS (TEWA DSS), is to provide operators with information related to the level of threat that aerial vehicles pose to own forces in a Surface-Based Air Defence (SBAD) scenario as well as to provide them with high-quality alternatives when weapon systems have to be assigned to engage aerial threats. The information provided by a TEWA DSS may then be used in conjunction with operator judgement, based on extensive knowledge, experience and training. Examples of such systems include the Battlefield Command Support System (SAAB BCSS, 2010) and Tactical Command and Control System (SAAB TCCS, 2010), developed by SAAB, as well as the GENESIS Ship Integrated Combat Management System, developed by the Turkish navy (Undersecretariat for Defence Industries, 2011). These “off-the-shelf” systems are typically available as “black box” systems in the defence industry. However, the problem is that the design rationales of such systems are usually not disclosed in the open literature, which limits the possibility of refining the software systems and fully understanding the working of the systems.

A TEWA DSS may typically be subdivided into two subsystems, known as a Threat Evaluation (TE) sub-system and a Weapon Assignment (WA) sub-system. The TE subsystem is responsible for evaluating the perceived level of threat that aerial vehicles pose to own forces (in terms of the notions of capability and intent), whereas the WA subsystem is responsible for proposing high-quality assignment proposals of available ground-based Weapon Systems (WSs) to engage aerial vehicles classified as threats. In 2007, Roux and Van Vuuren (2007), reviewed the state of the art of such TEWA DSSs about which information was available in the open literature at that stage. In 2008, they went further by suggesting a design for a complete, generic TE subsystem for use in a SBAD context (Roux and Van Vuuren, 2008). The aim in this paper is to build on the work of Roux and Van Vuuren by putting forward the design of a WA subsystem as part of the larger TEWA DSS. The design follows the structured approach proposed by Roux and Van Vuuren (2007) and contains a refinement of the substructures contained in such a subsystem.

The paper is structured as follows. The physical and functional elements of a TEWA DSS are described in §2 within the context of an SBAD environment. A detailed design of a generic WA subsystem counterpart to Roux and Van Vuuren’s TE subsystem design in (Roux and Van Vuuren, 2008) is put forward in §3. Various WS-assignment modelling paradigms are considered in §4 and the working of the proposed WA subsystem is illustrated in §5, by illucidating the chronological order of events which occur during the working of such a subsystem in the context of a simulated SBAD scenario. The numerical results of the various models are presented in §6 and discussed in §7. The paper closes with some conclusions in §8 and a discussion on possible ideas for future work in §9.

2 A TEWA DSS in a SBAD environment

A typical SBAD environment is concerned with Defended Assets (DAs) which require protection from aerial vehicles entering the three-dimensional space containing the DAs (also known as the defended airspace). A network of sensors is responsible for detecting these vehicles and protection is typically afforded by means of a number of pre-deployed ground-based WSs which are able to engage aerial vehicles. The collection of these hardware systems (i.e. the DAs, sensors and WSs) are known as the physical elements of the SBAD environment.

The problem of defending DAs is twofold. The first subproblem is to evaluate the perceived level of threat that aerial vehicles pose to DAs and the second subproblem is to assign available WSs to engage the aerial vehicles that are classified as threats. The second of these subproblems is more commonly known in the operations research literature as the WA problem (Manne, 1958; Ahuja, 2003) in which the assignments of WSs are proposed to engage threats based on a score achieved with respect to some pre-specified set of criteria (also known as objectives). The individual responsible for solving both of these subproblems in real-time is known as a Fire Control Officer (FCO). The FCO may easily be overwhelmed or mislead by vague and often contradicting information during combat situations, because (s)he has to make decisions
with respect to evaluating the levels of threat of aerial vehicles as well as the choice of which WSs to assign to threats under severely stressful conditions and over very short time horizons.

A computerised TEWA DSS may be employed to aid the FCO in these difficult decisions. Such a TEWA system forms part of the so-called functional elements (or processes) of an SBAD environment. Roux and Van Vuuren (2008) proposed a set of general functional elements for air defence decision support. These functional elements include Operator Decision Support, Testing and Training, Track Management (TM), MAP processing, Maintenance, a TEWA Data Manager, Attribute Management (AM), TE, Engagement Quantisation (EQ) and WA. These functional elements, as well as the flow of information between them, are illustrated schematically in Figure 2.1, and are discussed in some detail in the remainder of this section.

![Figure 2.1: SBAD functional elements and the flow of information between these elements (Roux and Van Vuuren, 2008).](image)

The first functional element in Figure 2.1 is operator decision support, which enables the FCO to communicate with the physical elements in the system. It may also be referred to as the link between man and machine and is typically implemented by means of a so-called Human Machine Interface (HMI), which provides the FCO with an interactive graphic user interface (Pannone, 2010; Ramkumar, 2012). The HMI displays the air picture on a screen and enables the FCO to view results generated by other functional elements (e.g. TE and WA results) as well as to perform decisions with respect to the evaluation and prioritisation of threats and the assignment of WSs for engaging aerial threats.
Military personnel have to be trained so that they are able to operate the functional elements of a SBAD system effectively. This may be achieved by testing and training procedures that are carried out during the pre-deployment stage of a mission, or at an earlier stage. The type of training required depends on the various capacities in which military personnel are deployed. Training procedures typically involve simulation functions which are responsible for imitating diverse SBAD scenarios (Roux and Van Vuuren, 2008). Testing usually involves evaluating the success of the integration of the system as an entity. This may be achieved by testing the performance of the system under controlled conditions, while analysing the results (Hower, 2006). During these testing procedures, scenarios may be employed that are similar to those generated for training purposes.

Typical information received from the sensor systems include so-called *measured (kinematic) attributes* of incoming aircraft, such as the speed or altitude at which they are travelling. These measured attributes are fused by a TM system in order to create a single so-called *system track* for each of the aircraft in the defended airspace (Roux and Van Vuuren, 2007). The process of TM also entails the classification of aircraft according to their platform type (e.g. *fixed* or *rotary wing* aircraft) as well as the classification of aircraft as *hostile, friendly* or *unknown*. Only those aircraft that are classified as hostile are considered for further evaluation with respect to the level of threat they pose to DAs.

Apart from creating system tracks and classifying aircraft, the TM system also includes a so-called *Flight Path Prediction* (FPP) module. The function of this module is to predict the flight paths of observed threats for a pre-specified number of future time stages. This is achieved by employing prediction models which utilise system track information from the TM. The output of the FPP is an array of predicted flight paths for each threat combined with a probability distribution for all of these paths, which is usually fused into a single, expected flight path for each threat. This single flight path is stored as part of the system track of each threat in the TEWA database, and serves as input to the TE and WA subsystems (Potgieter, 2012; Van Staden, 2012). The number of future time stages over which flight paths are predicted should be chosen carefully, since the statistical confidence associated with a predicted flight path diminishes as the number of future time stages increases.

MAP processing comprises the element of an SBAD system where the functionality and information related to geographical maps reside (Roux and Van Vuuren, 2008). It usually contains a so-called *MAP server* in which a database resides for storing map and other geographical information, a component for the calculation of *Line Of Sight* (LOS) attributes and a display component for planning and prioritisation.

During the pre-deployment stage, the various models residing in the TE and WA processes require certain initialisation parameters based on intelligence with respect to the aircraft types, modes of armament and attack profiles typically employed by the opposing force, as well as various WS parameter threshold values. A set of initialisation parameters may be required for each TE and WA model and these parameters may be updated during the mission by means of carefully established maintenance procedures.

A central functional element of the system is the TEWA data manager together with its database. This element serves as the communication hub of the system from where data are continually relayed between the physical and functional elements of the SBAD system. The data manager is also responsible for the storage of TEWA-related data in a so-called *TEWA database* as well as for the processing of data (e.g. sensor and system track processing).

The measured attributes of aircraft obtained from the sensor systems are used by an AM system to compute further so-called *derived attributes* associated with each system track (i.e. each threat) (Roux and Van Vuuren, 2008). An example of such an attribute is the acceleration of an aircraft. Once the derived attributes have been calculated, they are stored in real time as part of the system track of each threat in the TEWA database. The derived attributes are employed in the TE functional element.

The TE subsystem proposed by Roux and Van Vuuren (2008) comprises two substructures, known as the *Threat Evaluation Model* (TEM) component and the *Threat Evaluation Fusion Model* (TEFM) component. The former component employs the measured attributes from the sensors, the derived attributes from the AM system as well as pre-deployment data and initialisation parameters to estimate the level of threat that each of the aircraft in the defended airspace poses to the DAs. This may be achieved by means of the suite
of mathematical threat evaluation models functioning concurrently.

The TEM component proposed by Roux and Van Vuuren (2008) comprises three classes of TE models which are distinguished from one another by the level of complexity and sophistication of the models contained in each class. The three classes of models are, in increasing order of complexity, flagging models, deterministic models and stochastic models:

(1) The flagging models are a suite of binary models. These models are qualitative in nature and simply alert the operator, by flagging an aircraft, if sudden changes in the kinematic behaviour of the aircraft are observed or if the aircraft seem to be engaging in some kind of hostile behaviour. Hence, flagging models are not able to distinguish between different levels of threat posed by aircraft.

(2) Deterministic models, on the other hand, are quantitative models, each based on some measure of threat, such as the expected travel time to a DA or some course/bearing-related measure. Each model in this class takes the observed kinematic data of aircraft as well as the DA deployment data as input and generates as output a normalised threat value for each aircraft.

(3) The TE models exhibiting the highest level of sophistication are the quantitative, stochastic models. Such a model takes the observed kinematic data of aircraft, DA deployment data, enemy arsenal intelligence and doctrine as input, and generates as output a single threat value for each aircraft-DA pair. Such a probability-based threat value is typically an estimate of the probability that an aircraft will attack and/or kill a specific DA.

The results generated by the TEM component are fused together by the TEFM component to obtain a single threat value for each threat so as to arrive at a single prioritised list of threats (i.e. a list containing a threat value for each threat) with respect to the collection of DAs. This may be achieved by some Multi-Criteria Decision Analysis (MCDA) value function method. The single threat list is presented to the FCO as DS in real time.

The remaining functional elements are the EQ subsystem and the WA subsystem. Since EQ entails the quantisation of the effectiveness of an engagement of an aerial threat by a WS, the EQ subsystem is considered a WA-related component and is hence considered together with the WA subsystem in the next section.

3 Design of a generic WA subsystem

This section contains a description of two WA-related subsystems, namely the EQ subsystem and the WA subsystem. The former subsystem encapsulates the processes involved in the quantisation of the engagement efficiency values that WSs attain when they are assigned to engage threats, whereas the latter subsystem is concerned with how the output information obtained from the EQ subsystem is used in conjunction with information from other functional elements to propose the assignments of WSs to engage threats. The description of the WA subsystem includes a framework for the processes involved in such a subsystem as well as the proposal of four WA model classes that may be incorporated in such a subsystem. Examples of WA models that reside within each of these classes are also given.

3.1 The EQ subsystem

The EQ subsystem contains two subcomponents, known as the Physical Element Filter (PEF) component and the Engagement Efficiency Matrix (EEM) component, as illustrated schematically in Figure 3.2. The purpose of the EQ subsystem is to map a number of data sets required by the WA subsystem into a single set, and this is achieved by the PEF component. The PEF component takes as input a measure of the efficiency value that a WS is capable of attaining when engaging a threat. This measure is known as the Single Shot Hit Probability (SSHP) of the WS and is the probability that a single shot fired at a target will
hit that target under a given set of conditions (Denney, 1970). This probability value is typically estimated by the manufacturers of WSs and is a function of a three-dimensional SSHP volume (Potgieter, 2012). The shape and size of this volume depend largely on the characteristics of the type of WS employed as well as the characteristics of the related threat engaged by the WS.

In order to utilise the information contained in the SSHP volume in components such as the EEM and the WA subsystem, the volume is typically discretised. This may be achieved by superimposing a three-dimensional grid over the SSHP volume. Each cell in the grid is then assigned the probability value which corresponds to the partition in which it predominantly lies within the SSHP volume. The discretisation procedure yields a three-dimensional SSHP matrix.

The SSHP values of WSs typically do not take into account the effects of external elements, such as meteorological conditions (e.g. precipitation, wind strength and direction, or cloud cover) and terrain obstacles (i.e. LOS restrictions) on the efficiency of WSs. In order to make provision for constraints or masking effects imposed by these elements, the SSHP values may be filtered, in effect discounting for such elements. For each external element, a filter matrix may be applied to the SSHP matrix, multiplying each entry in the SSHP matrix by the corresponding entry in the filter matrix. The result of this application yields a so-called weapon system efficiency matrix (WSEM). In the event of applying more than one filter matrix, an additional
correlation matrix should also be applied to the SSHP matrix so as to avoid double filtering due to possible dependencies between certain environmental conditions.

The resulting WSEM produced as output by the PEF component is presented, in real-time, to the EEM component. The EEM component uses this matrix in conjunction with the PFPs from the TM system to generate as output a three-dimensional matrix known as the EEM. The EEM has as dimensions WSs, threats, and time stages and contains the filtered SSHP information of each WS with respect to each threat for the current as well as the predicted future time stages and is stored in the TEWA database. The EEM serves as partial input to the models incorporated in the WA subsystem, which are discussed in the next section.

3.2 The WA subsystem

The results generated by the WA subsystem serve as an essential part of the real-time DS provided to the FCO. In this section, we propose that the WA subsystem be designed to contain two sub-components, namely a Weapon Assignment Model (WAM) component and a Weapon Assignment Solution Selection (WASS) component, as illustrated schematically in Figure 3.2. The WAM component constitutes a collection of mathematical assignment models each responsible for solving a special variant of the WA problem by proposing assignments of WSs to threats in real-time. These assignments are typically based on their desirability with respect to some pre-specified objective function or set of objective functions, as dictated by the relevant WA problem variant. Furthermore, it is proposed that the WAM component constitute four classes of WA models, as illustrated schematically in Figure 3.1. Detailed descriptions of these model classes follow.

![Figure 3.1: Four classes of WA models proposed for inclusion in the WAM component: (1) single-objective, static WA models, (2) multi-objective, static WA models, (3) single-objective, dynamic WA models and (4) multi-objective, dynamic WA models. A grey-scaled colour scheme is used to denote model complexity, with darker colours corresponding to more complex WA models.](image)

3.2.1 Single-objective, static WA models in the WAM component

Single-objective, static WA models are static in a temporal sense, i.e. they are solved over a single time stage at a time (the WA problem is typically solved only for the current time stage or a specified time stage in the future). A further characteristic of models in this class is that they involve single-objective optimisation (i.e. only one objective function is included in such a model, which either has to be minimised or maximised).

One of the earliest examples of a single-objective, static WA model in the literature is the classical weapon target assignment problem by Manne (1958). In this formulation the aim is to assign or match $m$ WSs to $n$ threats in such a way that the accumulated expected survival probability of threats is minimised. A number of authors have contributed towards the refinement of this model. Notable examples include Matlin (1970) and Burr and Eckler (1972). In 2003, Ahuja et al. (2003) altered the original formulation of Manne (1958) by introducing a differentiation between the types of WSs employed in the model formulation. This formulation may be further adjusted by making provision for the assignment of at most $\kappa \geq 1$ WSs to any threat. This model extension is called the $\kappa$-WA problem and serves as the single-objective, static WA model example adopted for illustrative purposes in this section.
The objective in the $\kappa$-WA problem is to minimise the accumulated survival probabilities of all the threats, weighted by the priorities of eliminating the respective threats. The probability of survival of a specific threat is calculated as the product of the probabilities of surviving engagements by the separate WSs assigned to engage it (i.e. the assumption is made that the events of a threat surviving engagements by two different WSs are independent). Suppose $m(\tau)$ WSs are available for assignment to any of the $n(\tau)$ observed aircraft classified as threats at time stage $\tau$. Let $V_j(\tau)$ denote the priority of eliminating threat $j$ at time stage $\tau$ (this value is typically determined during the process of threat evaluation) and let $q_{ij}(\tau)$ denote the survival probability of threat $j$ if engaged by WS $i$ at time stage $\tau$. Note, therefore, that $q_{ij}(\tau) = 1 - p_{ij}(\tau)$, where $p_{ij}(\tau)$ denotes the probability of eliminating threat $j$ if it is engaged by WS $i$ during time stage $\tau$ (i.e. the SSHP value of WS $i$ with respect to threat $j$ at time stage $\tau$). Furthermore, denote the number of ammunition units available to WS $i$ by $A_i$ (these units may, for example, be bursts of cannon bullets or missiles). The decision variables in the formulation are binary in nature and are denoted by $x_{ij}(\tau)$, taking a value of 1 if WS $i$ is assigned to engage threat $j$ at time stage $\tau$, or the value 0 otherwise. The objective in the $\kappa$-WA problem is to

$$\min \sum_{j=1}^{n(\tau)} V_j(\tau) \prod_{i=1}^{m(\tau)} q_{ij}(\tau)^{x_{ij}(\tau)}$$

subject to the constraints

$$\sum_{j=1}^{m(\tau)} x_{ij}(\tau) \leq A_i, \quad i = 1, \ldots, m(\tau),$$

$$\sum_{i=1}^{n(\tau)} x_{ij}(\tau) \leq \kappa, \quad j = 1, \ldots, n(\tau),$$

$$x_{ij}(\tau) \in \{0, 1\}, \quad i = 1, \ldots, m(\tau),$$

$$j = 1, \ldots, n(\tau).$$

Constraint set (3.2) ensures that WS $i$ is assigned at most $A_i$ times for engagement, while constraint set (3.3) ensures that no more than $\kappa$ WSs are assigned to engage any threat. Finally, constraint set (3.4) enforces the binary nature of the decision variables.

### 3.2.2 Multi-objective, static WA models in the WAM component

Multi-objective, static WA models are similar to single-objective, static WA models in the sense that they are temporally static in nature. However, multi-objective, static models are distinguished from the class of models described in §3.2.1 in the sense that they involve multi-objective optimisation (i.e. more than one objective function has to be minimised or maximised). The objectives in such models are typically conflicting in nature, which requires that a trade-off be sought between objective function values. Multi-objective optimisation therefore naturally gives rise to the notion of Pareto optimality instead of the simpler and more intuitive notion of optimality. In this generalised framework of optimality a subset of solutions, known as the Pareto optimal set of solutions or the nondominated set of solutions, is sought which has the property of being superior to the remaining solutions in the solution space with respect to all the objectives, but being inferior to the solutions in the Pareto optimal set with respect to one or more, but not all, objectives (Deb and Srinivas, 1995).

In 2013, Lötter et al. (2013), derived a list of objectives in consultation with six military experts which may be used in the formulation of the multi-objective WA problem. They selected two objectives from this list for the formulation of a bi-objective, static WA model. The first objective was to minimise the accumulated survival probabilities of the threats in the system, weighted by the priorities of eliminating each of these threats (similar to the objective function in the $k$-WA model (3.1)–(3.4)), while the second objective was to minimise the accumulated monetary cost of assigning WSs to threats. Their model serves as a stepping stone.
for the tri-objective, static WA model adopted for illustrative purposes in this section. Our third objective is to maximise the number of times that a WS is available for engagement after the proposed assignment (in a bid to ensure that all weapons are reusable after the assignment). Adopting the same notation as in §3.2.1, the aim in this tri-objective WA model is to

\[
\text{minimise } \sum_{j=1}^{n(\tau)} V_j(\tau) \prod_{i=1}^{m(\tau)} q_{ij}(\tau)^{x_{ij}(\tau)} \\
\text{minimise } \sum_{i=1}^{m(\tau)} C_i \sum_{j=1}^{n(\tau)} x_{ij}(\tau) \\
\text{maximise } \min_i \left\{ A_i(\tau) - \sum_{j=1}^{n(\tau)} x_{ij}(\tau) \right\}
\]

subject to the constraints

\[
\sum_{j=1}^{n(\tau)} x_{ij}(\tau) \leq A_i, \quad i = 1, \ldots, m(\tau),
\]

\[
\sum_{i=1}^{m(\tau)} x_{ij}(\tau) \leq \kappa, \quad j = 1, \ldots, m(\tau),
\]

\[
x_{ij}(\tau) \in \{0, 1\}, \quad i = 1, \ldots, m(\tau), \quad j = 1, \ldots, n(\tau),
\]

where \(C_i\) denotes the cost in US dollars of firing a single burst of WS \(i\) ammunition and the meanings of the constraints are the same as in (3.2)–(3.4). The tri-objective model (3.5)–(3.10) is again formulated under the assumption that the events of a threat surviving engagements by two different WSs are independent.

3.2.3 Single-objective, dynamic WA models in the WAM component

Single-objective, dynamic WA models are similar to the class of WA models described in §3.2.1 in that they involve single-objective optimisation. However, they differ from that class of models in the sense that they take into account the change in the kinematic behaviour of aircraft over a pre-specified temporal interval or time horizon, which classifies them as dynamic WA models. The dynamic nature of these models imply that the WA problem is solved for the current time stage as well as for a number of pre-specified future time stages in the time continuum. These models therefore also involve a scheduling element of when WSs should engage threats in addition to the desirability or effectiveness of matching WSs to threats (the assignment element).

In 2013, Van der Merwe and Van Vuuren (2013) formulated the single-objective, dynamic WA problem as a vehicle routing problem with time windows (VRPTW). The VRPTW is a well-known combinatorial optimisation problem in the Operations Research literature in which the aim is to find an optimal fleet composition of vehicles as well as an optimal set of routes for these vehicles in order to serve a set of customers with known demands, based on a pre-specified objective (Toth and Vigo, 2001). In the VRPTW, each customer is associated with a service time duration (the total number of time stages required to serve the customer) as well as a service time window (a set of allowable contiguous time stages during which the customer has to be served). A central depot is included in the formulation from where each route is required to start and return. The depot is also associated with a time window, known as the scheduling horizon, during which vehicles are allowed to serve the customers. Vehicles are allowed to serve more than one customer on a specific route, but the time stage at which service of a specific customer ends plus the travel time of the vehicle to reach the next customer served along the route should not exceed the time stage
during which service commences at the next customer (i.e. services of consecutive customers by a single vehicle should not overlap in time). Furthermore, the total demand of customers on a specific route should not exceed the demand satisfying capacity of the vehicle assigned to that route. A typical objective in the VRPTW is to minimise the accumulated service cost, which is usually the total travel cost incurred by all the vehicles.

Van der Merwe and Van Vuuren (2013) proposed that, in the context of an SBAD system, the WSs be modelled as vehicles, the ammunition carried by WSs be modelled as commodities that have to be delivered to customers and the threats be modelled as customers. Hence, the scheduling over time of successive engagements by WSs may be thought of as scheduling vehicles (the WSs) delivering commodities (the ammunition) to customers (the aerial threats) as in the VRPTW. The capacity of each vehicle should be taken as the number of units of ammunition available to the relevant WS. Each threat is associated with a number of so-called engagement fire windows (FWs) analogous to the single time window of a customer in the VRPTW, specifying the first-time-to-fire (FTTF) and the last-time-to-fire (LTTF) for that threat. Each WS should be able to achieve a positive SSHP value with respect to the threat in question during each stage of each FW associated with the (WS, threat)-pair. It is therefore presumed that the set of FWs associated with a (WS, threat)-pair is induced by a combination of terrain surface masking, meteorological conditions, the position of the WS and the predicted flight path of the threat. As in the VRPTW, a virtual depot is included and may be seen as an artificial construct which represents an idle state during which no WS engages any threats. The system is required to start from the idle state and return to this state again after all threats have been engaged.

The model proposed by Van der Merwe and Van Vuuren (2013) serves as the single-objective, dynamic WA model example adopted for illustrative purposes in this section. In this model, the assumption is made that only one unit of ammunition is delivered to a customer at a specific time stage, which is taken as one burst of the ammunition of the WS. The same notation is used as in §3.2.1. In addition, the depot is indexed as threat 0 and threat \(n+1\), and has a demand of zero. Let \(d_i\) denote the number of time stages that WS \(i\) requires to “travel” between consecutive threats assigned to it. The value of \(d_i\) may be thought of as the setup time of WS \(i\) required for aiming at or tracking of a threat before an actual engagement may take place. Define a FTTF \(e_{ijk}\) and a LTTF \(\ell_{ijk}\) for WS \(i\) when engaging threat \(j\) during the pair’s \(k^{th}\) FW. Let the engagement time duration of threat \(j\) by WS \(i\) during the pair’s \(k^{th}\) FW be denoted by \(s_{ijk}\), where \(s_{ijk} = \ell_{ijk} - e_{ijk} + 1\), and let \(f_{ij}\) be the number of distinct FWs for (WS, threat)-pair \(ij\). Furthermore, let \(p_{ijk}\) denote the SSHP value associated with threat \(j\) if engaged by WS \(i\) during the \(k^{th}\) FW, and let \(q_{ijk} = 1 - p_{ijk}\).

In the model, the binary decision variable \(x_{ijk}\) takes the value 1 if WS \(i\) engages threat \(j\) during the \(k^{th}\) FW associated with the (WS, threat)-pair, or a value of 0 otherwise. A binary auxiliary variable \(y_{ihj}\) is also employed, which may be interpreted as a vehicle flow variable, and takes a value of 1 if threat \(h\) directly precedes threat \(j\) in a sequence of engagements by WS \(i\), or a value of 0 otherwise. The objective in the single-objective, dynamic WA is to

\[
\text{minimise } \sum_{j=1}^{n} V_j \prod_{i=1}^{m} \prod_{k=1}^{f_{ij}} (q_{ijk})^{x_{ijk}}
\]  

(3.11)
3 Design of a generic WA subsystem

subject to the constraints

\[
\sum_{i=1}^{m} \sum_{h=0}^{n+1} y_{ihj} \leq \kappa, \quad j = 1, \ldots, n, \quad (3.12)
\]
\[
\sum_{h=1}^{n+1} y_{ihj} = 1, \quad i = 1, \ldots, m, \quad (3.13)
\]
\[
\sum_{h=1}^{n+1} y_{ih0} = 1, \quad i = 1, \ldots, m, \quad (3.14)
\]
\[
\sum_{h=0}^{n+1} y_{ihj} - \sum_{h=0}^{n+1} y_{ijh} = 0, \quad i = 1, \ldots, m, \quad j = 0, \ldots, n + 1, \quad (3.15)
\]
\[
\sum_{k=1}^{n+1} x_{ijk} = \sum_{h=1}^{n+1} y_{ihj}, \quad i = 1, \ldots, m, \quad j = 1, \ldots, n, \quad (3.16)
\]

\[
\sum_{k=1}^{n+1} (e_{ikh} + s_{ikh}) x_{ikh} - \sum_{k=1}^{n+1} e_{ijk} x_{ijk} + d_i < (1 - y_{ihj})L, \quad i = 1, \ldots, m, \quad j = 1, \ldots, n, \quad h = 1, \ldots, n, \quad (3.17)
\]
\[
\sum_{j=1}^{n} \sum_{k=1}^{n+1} x_{ijk} \leq A_i, \quad i = 1, \ldots, m, \quad (3.18)
\]
\[
y_{ihj} \in \{0, 1\}, \quad i = 1, \ldots, m, \quad j = 0, \ldots, n + 1, \quad h = 0, \ldots, n + 1, \quad (3.19)
\]
\[
x_{ijk} \in \{0, 1\}, \quad i = 1, \ldots, m, \quad j = 0, \ldots, n + 1, \quad k = 1, \ldots, f_{ij}. \quad (3.20)
\]

Constraint set (3.12) ensures that at most \(\kappa\) WSs are assigned to engage any threat during the scheduling horizon. Constraint set (3.13) ensures that WS \(i\) “leaves the depot” (idle state) exactly once, if it is assigned to engage threats at all, while constraint set (3.14) ensures that WS \(i\) “returns to the depot” exactly once after being used to engage threats. Constraint set (3.15) ensures, if a threat is serviced by WS \(i\), that the WS “leaves the threat” again in order to “move on” to engage the next threat assigned to it. Constraint set (3.16) ensures, if threat \(h\) precedes threat \(j\) for engagement by WS \(i\), that threat \(h\) is engaged during exactly one stage. If threat \(h\) is engaged directly before threat \(j\) by WS \(i\), constraint set (3.17) ensures that the time stage during which the engagement of threat \(h\) starts plus the time it takes to engage threat \(h\) plus the time it takes for WS \(i\) to “travel” from threat \(h\) to threat \(j\) does not exceed the time stage during which engagement of threat \(j\) starts, where \(L\) is a large number. Furthermore, constraint set (3.17) also ensures that the stage during which WS \(i\) engages threat \(j\) is within a FW associated with the (WS, threat)-pair. Constraint set (3.18) ensures that the capacity of WS \(i\) is not exceeded and finally, constraint sets (3.19)–(3.20) ensure the binary nature of the decision and auxiliary variables. Note that the WA model (3.12)–(3.20) conforms to the standard VRPTW formulation in the literature.

3.2.4 Multiobjective, dynamic WA models in the WAM component

The final class of WA models comprises the most complex WA models of the four model classes proposed in this paper. They are multi-objective, dynamic WA models which reside within the black quadrant in Figure 3.1. These models involve solving the WA problem in a multi-objective context over a number of
pre-specified, predicted time stages and also include a scheduling element similar to models in the class of single-objective, dynamic WA models (described in §3.2.3), i.e. requiring decisions as to when WSs should engage threats in addition to which WSs should engage which threats. Although the models in this class are limited in the military operational research literature, one possibility for such a model is to formulate the WA problem as a variant of the VRPTW, but with multiple objectives (in other words, a composition of the WA model examples from §3.2.2 and §3.2.3). Numerous examples of such multi-objective VRPTWs exist in the operations research literature, including the formulations of Kallehauge et al. (2005), Geiger (2008) and Jozefowiez et al. (2008).

3.2.5 WAM component implementation suggestions

It is important not to overwhelm the FCO with information when providing DS during combat situations, since unnecessary information may cause confusion when important WA engagement decisions have to be made in severely stressful situations within very short timeframes. It is therefore proposed that during the predeployment stages of a mission, the FCO uses a decision tree, such as the one shown in Figure 3.2, to configure the actual WAMs for inclusion in the system.

In particular, it is proposed that the FCO should be able to decide whether the models to include in the WAM should be of a static or dynamic nature first, after which he should then be able to specify the inclusion of a single or multiple objectives in the models. The FCO should then be able to configure one or more WAMs (from the same class) by choosing objectives from a list of possible objectives. An example of such a list is shown in Table 3.1. The objectives available to the FCO to choose from should be dictated by the availability and quality of data. The FCO should be able to perform (and alter) these configuration decisions easily (e.g. by means of radio buttons via the HMI). A default WAM should also be specified during the design phase of the DSS to accommodate the case where the FCO does not choose any WAM.

<table>
<thead>
<tr>
<th>No</th>
<th>Objective</th>
<th>Possible constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Minimise aggregated survivability of observed aerial threats as a result of assignment, weighted by the priorities of eliminating the threats</td>
<td>Maximum number of WSs to assign</td>
</tr>
<tr>
<td>2</td>
<td>Minimise aggregated cost of assignment</td>
<td>Budget threshold on accumulated assignment cost</td>
</tr>
<tr>
<td>3</td>
<td>Minimise the maximum engagement time in the assignment</td>
<td>Maximum length threshold for FWs or minimum SSHP threshold within FWs</td>
</tr>
<tr>
<td>4</td>
<td>Maximise minimum number of times a WA can re-engage after the assignment</td>
<td>Number of available ammunition rounds at WSs</td>
</tr>
</tbody>
</table>

Table 3.1: A list of possible objectives and constraints from which to configure WA models to employ in the four classes in Figure 3.1.

In the event that the FCO chooses to include dynamic WA models in the WAM component, it is further proposed that the FCO should be able to specify the number of time stages to include in the prediction window of the threats. A default number of time stages to include in the prediction window should also be set during the design phase of the DSS to accommodate the case where the FCO does not specify a prediction window. The implication that the prediction window has on the WAM component is that for the static classes of WAMs (i.e. the models described in §3.2.1 and §3.2.2) the WA problem is solved separately for each stage in the specified prediction window taking into account the successful elimination of threats during previous stages and for the dynamic class of WAMs (i.e. the models described in §3.2.3 and §3.2.4), the WA problem is solved over the entire prediction window.

It is also proposed that for each pre-configured WAM there should be a collection of solution methodologies available for solving the WA problem. The FCO’s preferred solution methodologies should be configurable during the pre-deployment stage. The idea is that when the FCO chooses a specific WAM, the WA problem
should be solved (approximately) by a collection of solution methodologies simultaneously. The results obtained by the various solution methodologies should be presented to the WASS component for further analysis.

Finally, time is a critical factor in providing efficient TEWA DS to the FCO. It is therefore important that the solution methodologies employed in solving the models in the WAM component should be able to solve the models almost instantaneously, albeit only approximately.

### 3.3 The WASS component

The final component in the WA subsystem is the WASS component. The objective of this component is to combine and filter the results presented by the WAM component and to provide the filtered results in an
4 The chronological order of TEWA events

The chronological order of TEWA events

The components described in the previous sections function as part of a so-called *TEWA cycle*. A trigger of such a cycle initiates consecutive calls to these components in an orderly fashion so as to provide real-time DS to the FCO. For the purposes of this paper, it is assumed that a TEWA cycle is triggered by a significant change in any of the data fields\(^2\) in the TEWA database or by the natural continuation of an implementation clock cycle. The sequential order in which the components function within a TEWA cycle is illustrated graphically in Figure 4.1.

Once a TEWA cycle is triggered, a snapshot of the TEWA database is taken at that specific time stage. This snapshot contains the data to be used during the current TEWA computation cycle. After the snapshot of the database has been taken, the AM commences operation and the measured attributes obtained from the sensor systems are fused by the TM, as described in §2, after which derived attributes are computed for each of the observed threats.

Upon completion of the computations by the AM, the TEM process is initiated, during which the TEMs described in §2 evaluate the perceived level of threat posed by the observed aircraft with respect to the DAs. Threat lists are computed containing the prioritised threat values and these lists are fused together by using an MCDA method\(^3\) within the TEFM component to obtain a single prioritised list of threats. This single list is stored in the TEWA database and presented to the FCO via the HMI. Once the results from the TE subsystem have been stored in the TEWA database, WA subsystem operation commences.

Another snapshot of the TEWA database is taken to acquire the additional data required by the WA components. The efficiency values that WSs are expected to achieve with respect to threats during future time stages are discretised and discounted for meteorological conditions and terrain obstacles within the EQ component, as described in §3.1. The output (i.e. an EEM) is stored in the TEWA database for each future time stage. Thereafter the WAM component employs the threat list produced by the TEFM component and the EEM produced by the EQ component to propose assignments of available WSs to engage the aerial threats. The outputs of the WAM component (i.e. a number of proposed WA lists) are stored in the TEWA database and presented to the WASS component in which the WA lists are filtered and presented to the FCO via the HMI in a manner configured by the operator during pre-deployment, as described in §3.3. The presentation of these results to the FCO marks the end of a full TEWA cycle.
Although the TEWA system may seem to function as a fully automated software system, it is important to acknowledge that the FCO should have the authority to override assignment decisions suggested by the system. He should, for example, be able to configure the TEMs actually used in the threat evaluation process, alter the prioritised threat list obtained from the TEFM component and alter the WA lists proposed by the WAM component, based on his experience and subjective judgement.

5 A realistic SBAD test scenario

The working of the WA subsystem described above is illustrated by applying it in the context of a simulated, but realistic, hypothetical SBAD scenario which was designed in consultation with a retired military expert (Visser, 2008). The scenario mimics a typical SBAD deployment where DAs based on a land surface require protection from approaching aerial threats by means of pre-deployed ground-based WSs. A top view of the scenario is shown in Figure 5.1. The physical elements of the scenario include two DAs, which are represented by the two black squares labelled DA\textsubscript{1} and DA\textsubscript{2}, twelve WSs consisting of eight Very SHOrt-Range Air Defence Systems (VSHORADSs) labelled V\textsubscript{1},\ldots,V\textsubscript{8} and four Close-In Weapon Systems (CIWSs) labelled C\textsubscript{1},C\textsubscript{2},C\textsubscript{3} and C\textsubscript{4}. The WSs are deployed symmetrically in two concentric circles around the DAs. Five aircraft labelled T\textsubscript{1},T\textsubscript{2},T\textsubscript{3},T\textsubscript{4} and T\textsubscript{5} approach the DAs in three synchronised groupings from different directions. Threats T\textsubscript{1} and T\textsubscript{2} form the first group and enters from a north-westerly direction to attack DA\textsubscript{2} by performing a so-called pitch-and-dive attack technique\textsuperscript{4} in formation. Threats T\textsubscript{3} and T\textsubscript{4} form the second group and act as decoys to the system. They enter from a south-westerly direction and fly in a straight line exactly over the DAs at high altitude to exit the system in a north-easterly direction without attacking any DAs. The final group consists only of threat T\textsubscript{5} which enters from the south to attack DA\textsubscript{2} by performing a
so-called *toss bomb* attack technique\(^5\).

Figure 5.1: Top view of a hypothetical SBAD scenario comprising two DAs, twelve WSs, five aerial threats and 120 time stages, together with indications of the locations of the threats \(T_1, T_2, T_3, T_4\) and \(T_5\) during time stage 39.

The time continuum of the scenario is subdivided into 120 four-second time stages indexed by the parameter \(\tau\). It is assumed that the TE process related to the scenario has been completed and that priority values for eliminating each threat during each time stage have been established. In addition, it is assumed that an EEM for each time stage has been computed, as described in §3.1. For the sake of simplicity, constraints imposed by terrain obstacles and weather conditions are ignored and hence the EEMs contain only the SSHP values that WSs are expected to achieve with respect to threats.

Furthermore, the cost of assigning a WS to a threat is assumed to be the cost of a unit of ammunition of the WS. The cost of a unit of ammunition for a VSHORADS is taken as the cost of a single surface-to-air missile (approximately US $100,000), while the cost of a CIWS is taken as the cost of a burst of cannon bullets (approximately US $3,400). These costs were realistic in 2011. Finally, the ammunition available to each WS is shown in Table 5.1.

<table>
<thead>
<tr>
<th>WS</th>
<th>(A_1)</th>
<th>(A_2)</th>
<th>(A_3)</th>
<th>(A_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V_1)</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>(V_2)</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>(V_3)</td>
<td>5</td>
<td>3</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>(V_4)</td>
<td>4</td>
<td>3</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 5.1: The ammunition available to each WS for the hypothetical SBAD scenario depicted in Figure 5.1.

### 6 Illustrative numerical results for the SBAD test scenario

Various solution methodologies may be employed to solve the WA model examples of §3 (approximately). This section contains a summary of the numerical results obtained by solving each of these models in the context of the SBAD scenario described in §5, as well as short descriptions of the solution methodologies employed in solving each model.
6 Illustrative numerical results for the SBAD test scenario

6.1 Results obtained from the single-objective, static WA model

Since the static WA models described in §3.2.1 and §3.2.2 consider only a single time stage at a time, these models are solved only over a single stage for the sake of brevity. Time stage 39 was chosen for illustrative purposes because of its diversity and the close proximity of the threat locations to the DAs when \( \tau = 39 \).

The EEM and the threat list corresponding to this time stage are shown in Table 6.1. The value of \( \kappa = 3 \) was assumed.

<table>
<thead>
<tr>
<th>WS</th>
<th>( T_1 )</th>
<th>( T_2 )</th>
<th>( T_3 )</th>
<th>( T_4 )</th>
<th>( T_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_1 )</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>( V_2 )</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>( V_3 )</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>( V_4 )</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( V_5 )</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( V_6 )</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( V_7 )</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( V_8 )</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( C_1 )</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>( C_3 )</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( C_4 )</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 6.1: (a) The EEM and (b) the threat list corresponding to time stage \( \tau = 39 \) for the hypothetical SBAD scenario depicted in Figure 5.1.

Due to the nonconvexity of the models in §3, it was decided to solve the single-objective, static WA model (3.1)–(3.4) approximately using a genetic algorithm\(^6\). Solutions to this model were represented as binary matrices of dimensions \( m(\tau) \times n(\tau) \) (recall that \( m(\tau) \) represents the number of WSs during stage \( \tau \) and that \( n(\tau) \) represents the number of threats in the system during stage \( \tau \)), where a value of 1 in row \( i \) and column \( j \) of the matrix indicates an assignment of WS \( i \) to threat \( j \), while a value of 0 indicates no assignment. The algorithm was initiated by generating an initial population of 200 candidate solutions randomly. The current population of solutions was then used to populate the next generation of solutions during each iteration. This was achieved by implementing a tournament selection procedure\(^7\) with pool and tour sizes of 2 and 100, respectively, to select parent solutions in an elitist manner from the current solution and to create offspring solutions by applying conventional crossover\(^8\) and mutation\(^9\) operators to the selected parent solutions. A mutation probability of 0.025 was used and the algorithm was iterated until a stopping criterion of 400 iterations was reached.

The best solution obtained from the WA model (3.1)–(3.4) is shown in Figure 6.2. In this solution WSs \( V_2 \), \( V_3 \) and \( C_1 \) are assigned to threat \( T_1 \), WS \( C_2 \) is assigned to threat \( T_2 \), WS \( V_1 \) is assigned to threat \( T_3 \) and WSs \( V_6 \) and \( V_7 \) are assigned to threat \( T_5 \). Note that threat \( T_4 \) is assigned no WSs. This assignment yields a priority-weighted accumulated survival probability value of the threats of 1.9628 at an assignment cost of US $506 800.

6.2 Results obtained from the multi-objective, static WA model

The multi-objective, static WA model (3.5)–(3.10) was again solved \( \kappa = 3 \) and for stage \( \tau = 39 \) of the simulated scenario described in §5, but this time using a multi-objective variant of the genetic algorithm described in §6.1, called the Nondominated Sorting Genetic Algorithm II (NSGA II)\(^10\), which is due to Agarwal et al. (2002). Solutions were again represented as binary matrices, as described in §6.1. The current population of solutions were ranked based on the nondominated statuses of individuals, as discussed in §3.2.2, and divided into nondominated fronts of solutions based on their rank values. This was achieved by implementing the Fast Nondominated Sorting Algorithm\(^11\) of Agarwal et al. (2002).
### Illustrative numerical results for the SBAD test scenario

<table>
<thead>
<tr>
<th>Threat</th>
<th>Priority</th>
<th>WSs assigned</th>
<th>Survival values</th>
<th>Prioritised survival probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$</td>
<td>0.99</td>
<td>$C_1, V_2, V_3$</td>
<td>0.8, 0.3, 0.9</td>
<td>$0.99 \times 0.8 \times 0.3 \times 0.9$</td>
</tr>
<tr>
<td>$T_2$</td>
<td>1.00</td>
<td>$C_2$</td>
<td>0.1</td>
<td>$1.00 \times 0.1$</td>
</tr>
<tr>
<td>$T_3$</td>
<td>0.76</td>
<td>$V_1$</td>
<td>0.9</td>
<td>$0.76 \times 0.9$</td>
</tr>
<tr>
<td>$T_4$</td>
<td>0.74</td>
<td>None</td>
<td>–</td>
<td>$0.74 \times 1$</td>
</tr>
<tr>
<td>$T_5$</td>
<td>0.50</td>
<td>$V_6, V_7$</td>
<td>0.9, 0.5</td>
<td>$0.5 \times 0.9 \times 0.5$</td>
</tr>
<tr>
<td>Total:</td>
<td></td>
<td></td>
<td></td>
<td>$1.9628$</td>
</tr>
</tbody>
</table>

#### Figure 6.1: (a) WS to threat assignment list and (b) top-view graphical illustration of the assignments of WSs to threats as proposed by the single-objective, static WA model (3.1)–(3.4) during stage $\tau = 39$ of the simulated scenario described in §5.

The current population was used to generate an intermediate population by using a tournament selection procedure\(^{12}\) to select parent solutions and performing crossover\(^{8}\) and mutation\(^{9}\) operators on the selected parent solutions. The solutions from the current and intermediate populations were then combined to form a larger population. The larger population was ranked and sorted using the Fast Nondominated Sorting algorithm and the next generation of candidate solutions was populated by first including the solutions with rank value 1, followed by the solutions with rank value 2 and so on until the size of the initial population is reached. The algorithm was initiated by generating an initial population of 200 candidate solutions randomly and was iterated until a stopping criterion of 400 iterations was reached.

The front of (nondominated) approximately Pareto optimal solutions obtained by the NSGA II is presented in Figure 6.2. It is clear from this figure that a good spread of solutions is obtained along the front and the trade-offs between the three objectives are clearly illustrated in the figure. The solutions achieve objective function values in the priority-weighted accumulated survival probabilities objective (3.5) and assignment cost objective (3.6) as indicated on the horizontal and vertical axes of the figure, respectively. The objective function values achieved for the re-engagability objective (3.7) may be interpreted as follows: The subset of solutions depicted in grey represent solutions in which the least re-engagable WS can engage twice after the assignment, while the subset of solutions depicted in black represents solutions in which the least re-engagable WS can engage three times after the assignment.

Suppose, for illustrative purposes, that an assignment is sought in which the least re-engagable WS can engage at least three times after the engagement and that a restriction of US $250,000 is placed on the accumulated assignment cost (as illustrated by the horizontal line labelled B in Figure 6.2). The best solution satisfying

---

\(^{12}\) Tournament selection is a method for selecting parent solutions from a population of candidate solutions.

\(^{8}\) Crossover is a genetic algorithm operator that combines the genetic information of two parent solutions to create new offspring.

\(^{9}\) Mutation is a genetic algorithm operator that alters a single solution in the population by changing one or more of its components.

---
both these criteria is the solution labelled C in the figure. In this solution, WS $V_2$ is assigned to threat $T_1$, WS $C_2$ is assigned to threat $T_2$ and WS $V_1$ is assigned to threat $T_3$. No WSs are assigned to threats $T_4$ or $T_5$. These assignments yield a priority-weighted accumulated survival probability value of the threats of 2.321 at an assignment cost of US $203,400. A graphical illustration of the assignments embodied in Solution C is shown in Figure 6.3.

Note that the solution to the single-objective, static WA model obtained in the previous section (see Figure 6.1) is also (approximately) Pareto-optimal for the multi-objective, static WA model; this solution is labelled A in Figure 6.2.

### 6.3 Results obtained from the single-objective, dynamic WA model

The single-objective, dynamic WA model (3.11)–(3.20) requires the earliest- and latest stage during which a WS may engage a threat (i.e. a FW for each (WS,threat)-pair) as well as the survival probability values of the threats when WSs are assigned to them during the FW period. The minimum length of a FW (including the WS set-up time required) was taken as four stages for all the WSs in the scenario. The earliest stage and latest stage during which a WS may engage a threat for each (WS,threat)-pair is shown in Table 6.2. The survival probability values of the threats were calculated using a fixed mean approach and these values are shown in Table 6.3.

Van der Merwe and Van Vuuren (2013) experimented with solving the single-objective, dynamic WA model (3.11)–(3.20) using a Simulated Annealing algorithm. Solutions to the single-objective, dynamic WA model (3.11)–(3.20) were represented as integer matrices of dimensions $m(\tau) \times n(\tau)$. If a WS is assigned to a threat, the corresponding entry in the solution matrix is an integer value representing the FTTF for
6 Illustrative numerical results for the SBAD test scenario

<table>
<thead>
<tr>
<th>Threat</th>
<th>Priority</th>
<th>WSs assigned</th>
<th>Survival values</th>
<th>Prioritised survival probability</th>
<th>Accumulated cost in US $</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$</td>
<td>0.99</td>
<td>$V_2$</td>
<td>0.3</td>
<td>$0.99 \times 0.3$</td>
<td>$1 \times 100,000$</td>
</tr>
<tr>
<td>$T_2$</td>
<td>1.00</td>
<td>$C_2$</td>
<td>0.1</td>
<td>$1.00 \times 0.1$</td>
<td>$1 \times 3,400$</td>
</tr>
<tr>
<td>$T_3$</td>
<td>0.76</td>
<td>$V_1$</td>
<td>0.9</td>
<td>$0.76 \times 0.9$</td>
<td>$1 \times 100,000$</td>
</tr>
<tr>
<td>$T_4$</td>
<td>0.74</td>
<td>None</td>
<td>–</td>
<td>$0.74 \times 1$</td>
<td>–</td>
</tr>
<tr>
<td>$T_5$</td>
<td>0.50</td>
<td>None</td>
<td>–</td>
<td>$0.5 \times 1$</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>Totals:</strong></td>
<td><strong>2.321 234 000</strong></td>
</tr>
</tbody>
</table>

Figure 6.3: Top-view graphical illustration of the assignments of WSs to threats in the Solution labelled C in Figure 6.2 to the multi-objective, static WA model (3.5)–(3.10).

<table>
<thead>
<tr>
<th>WS</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$T_3$</th>
<th>$T_4$</th>
<th>$T_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_1$</td>
<td>72</td>
<td>72</td>
<td>32</td>
<td>33</td>
<td>86</td>
</tr>
<tr>
<td>$V_2$</td>
<td>0</td>
<td>0</td>
<td>35</td>
<td>33</td>
<td>98</td>
</tr>
<tr>
<td>$V_3$</td>
<td>34</td>
<td>36</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$V_4$</td>
<td>19</td>
<td>18</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$V_5$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$V_6$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$V_7$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>34</td>
</tr>
<tr>
<td>$V_8$</td>
<td>83</td>
<td>80</td>
<td>0</td>
<td>0</td>
<td>57</td>
</tr>
<tr>
<td>$C_1$</td>
<td>66</td>
<td>64</td>
<td>19</td>
<td>17</td>
<td>85</td>
</tr>
<tr>
<td>$C_2$</td>
<td>64</td>
<td>52</td>
<td>0</td>
<td>19</td>
<td>0</td>
</tr>
<tr>
<td>$C_3$</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$C_4$</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 6.2: (a) The earliest stage and (b) the latest stage during which a WS may engage a threat.

the (WS,threat)-pair in question. Otherwise, the entry is zero, indicating that the (WS,threat)-pair is not assigned. The algorithm was initiated by generating an initial feasible solution in a greedy fashion. This solution was stored as the incumbent solution. During each iteration of the algorithm, a neighbourhood move operator performed a number of intelligent moves randomly in order to obtain neighbouring solutions. The temperature which controls the randomness of the search in the algorithm was lowered by implementing a geometric cooling schedule with an initial temperature of 0.1 and a cooling factor of 0.01. The algorithm
was iterated for 9 iterations or until the temperature reached a stopping temperature of 0.01. The incumbent solution\(^1\) was taken as an approximate solution.

The following assignments are proposed by the model for \(\kappa = 2\): WSs \(V_3\) and \(C_2\) are assigned to Threat \(T_1\) at time stages 34 and 64, respectively, while WSs \(C_1\) and \(C_2\) are assigned to threat \(T_2\) at time stages 64 and 52, respectively. Similarly, WSs \(V_1\) and \(C_1\) are assigned to threat \(T_3\) at time stages 32 and 19, respectively, while WSs \(C_3\) and \(C_4\) are assigned to threat \(T_4\) at time stages 1 and 4, respectively. Finally, WSs \(V_8\) and \(C_1\) are assigned to threat \(T_5\) at time stages 57 and 85, respectively. These assignments yield a priority weighted accumulated survival probability value\(^2\) of the threats of 0.1595 at an accumulated assignment cost of US\$323 800. These assignments appear in Table 6.4 and a graphical illustration of the assignments is shown in Figure 6.4. The dotted lines in the figure represent the predicted flight path of the threats and the bold lines represent the FWs for each threat. The underlying SSHP values of each WS are illustrated by means of a grey-scaled colour scheme.

### Table 6.3: The fixed mean survival value of threats for each (WS,threat)-pair over the FWs presented in Table 6.2.

<table>
<thead>
<tr>
<th>WS</th>
<th>(T_1)</th>
<th>(T_2)</th>
<th>(T_3)</th>
<th>(T_4)</th>
<th>(T_5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V_1)</td>
<td>0.451</td>
<td>0.451</td>
<td>0.367</td>
<td>0.369</td>
<td>0.481</td>
</tr>
<tr>
<td>(V_2)</td>
<td>1.000</td>
<td>1.000</td>
<td>0.374</td>
<td>0.369</td>
<td>0.435</td>
</tr>
<tr>
<td>(V_3)</td>
<td>0.371</td>
<td>0.338</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>(V_4)</td>
<td>0.340</td>
<td>0.328</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>(V_5)</td>
<td>1.000</td>
<td>1.000</td>
<td>0.701</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>(V_6)</td>
<td>0.474</td>
<td>0.468</td>
<td>1.000</td>
<td>1.000</td>
<td>0.420</td>
</tr>
<tr>
<td>(V_7)</td>
<td>0.192</td>
<td>0.156</td>
<td>1.000</td>
<td>0.560</td>
<td>1.000</td>
</tr>
<tr>
<td>(V_8)</td>
<td>1.000</td>
<td>1.000</td>
<td>0.390</td>
<td>0.238</td>
<td>1.000</td>
</tr>
<tr>
<td>(C_1)</td>
<td>1.000</td>
<td>1.000</td>
<td>0.075</td>
<td>0.207</td>
<td>1.000</td>
</tr>
</tbody>
</table>

### Table 6.4: WS to threat assignment list as well as the FTTF and LTTF as proposed by the single-objective, dynamic WA model (3.11)–(3.20).

<table>
<thead>
<tr>
<th>Threat</th>
<th>WSs assigned</th>
<th>FTTF</th>
<th>LTTF</th>
<th>Accumulated cost in US $</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_1)</td>
<td>(V_3)</td>
<td>34</td>
<td>56</td>
<td>(1 \times 100) 000</td>
</tr>
<tr>
<td></td>
<td>(C_2)</td>
<td>64</td>
<td>70</td>
<td>(1 \times 3) 400</td>
</tr>
<tr>
<td>(T_2)</td>
<td>(C_1)</td>
<td>64</td>
<td>81</td>
<td>(1 \times 3) 400</td>
</tr>
<tr>
<td></td>
<td>(C_2)</td>
<td>52</td>
<td>69</td>
<td>(1 \times 3) 400</td>
</tr>
<tr>
<td>(T_3)</td>
<td>(V_1)</td>
<td>32</td>
<td>63</td>
<td>(1 \times 100) 000</td>
</tr>
<tr>
<td></td>
<td>(C_1)</td>
<td>19</td>
<td>40</td>
<td>(1 \times 3) 400</td>
</tr>
<tr>
<td>(T_4)</td>
<td>(C_3)</td>
<td>1</td>
<td>19</td>
<td>(1 \times 3) 400</td>
</tr>
<tr>
<td></td>
<td>(C_4)</td>
<td>4</td>
<td>23</td>
<td>(1 \times 3) 400</td>
</tr>
<tr>
<td>(T_5)</td>
<td>(V_8)</td>
<td>57</td>
<td>85</td>
<td>(1 \times 100) 000</td>
</tr>
<tr>
<td></td>
<td>(C_1)</td>
<td>85</td>
<td>103</td>
<td>(1 \times 3) 400</td>
</tr>
</tbody>
</table>

| Total: |               |      |      | \(323\) 800               |
7 Discussion of results

The single-objective, static WA model (3.1)–(3.4) is easy to implement and can be solved (approximately) almost instantaneously by using a standard genetic algorithmic approach. This model is able to propose high-quality assignments of WSs to threats, but the model is limited to solving the WA problem for only a single time stage at a time. Hence, in the context of an SBAD scenario, such as the scenario described in §5, the model has to be solved separately for each time stage in the time continuum, taking into account the elimination of threats during previous time stages. Furthermore, this model is only able to present the FCO with a single solution for any given time stage. Hence, the model only provides a limited selection of options from which the FCO may choose. Finally, the fact that the model is solved for a single stage at a time (and implying that assignments have to made during the specific time stage in question) may lead to achieving only locally optimal solutions. Incorporating a time element where future time stages are also considered in the model, may lead to better solutions, since it may be more favorable to wait until future time stages before proposing assignments of WSs to threats when WSs may achieve larger SSHP values with respect to these threats.

The multi-objective, static model (3.5)–(3.10) is also able to propose high-quality assignments of WSs to threats and is fairly easy to implement and solve by means of the NSGA II (Agarwal et al., 2002). The main advantage of using this model, is that it is able to provide the FCO with a set of Pareto optimal solutions which represent good trade-offs between various assignment objectives (as illustrated in Figure 6.2) from which he may choose one to implement. Although it is possible that the FCO may be overwhelmed by the solutions in the Pareto optimal set when this set is large and assignment decisions have to be made very rapidly, this problem may be mitigated by careful incorporation of aspiration criteria into a DSS (for example, by constraining one or more of the objectives so as to narrow down the set of feasible solutions in the Pareto optimal set). Finally, this model is again only able to provide solutions for a single time stage at a time as is the case with the single-objective, static WA model (3.1)–(3.4), which may again lead to achieving only locally optimal solutions.

The single-objective, dynamic model (3.11)–(3.20) is the hardest to implement of the three classes of WA models illustrated in this paper. However, the dynamic nature of this model is its main advantage — it can solve the WA problem during any time stage over a specified number of future time stages, providing

Figure 6.4: Top-view graphical illustration of the assignments of WSs to threats proposed by the single-objective, dynamic WA model (3.11)–(3.20) for a prediction period of 120 seconds.
the FCO with approximately optimal fire windows during which to assign WSs to threats. Although the 
model is able to provide the FCO with high-quality assignments and corresponding time schedules for these 
assignments, it is only able to generate a single best solution during each implementation run (similar to the 
results generated by the single-objective, static WA model (3.1)–(3.4)) for the prediction window in question, 
rather than a selection of nondominated solutions from which the FCO may choose, as is the case with the 
multi-objective, static WA model (3.5)–(3.10).

8 Conclusion

The problem of WA decision support in an SBAD environment was considered in this paper. A brief 
review of the available literature on TEWA DSS in an SBAD environment was given to serve as a point 
of departure. A design framework for a first-order generic WA subsystem was proposed. This included a 
detailed description of the substructures of such a subsystem. In addition, four classes of WA models of 
different levels of complexity were also proposed for inclusion in the WAM component of the WA subsystem. 
This component serves as the core of the WA subsystem as was emphasized by including detailed worked 
examples for model prototypes in three of the proposed model classes in the context of a simulated, but 
realistic SBAD scenario. It was found that the trio of model prototypes are able to provide high-quality, 
realistic assignments of WSs to threats. The advantages and disadvantages of employing members of the 
various model classes were finally highlighted.

9 Future work

Although the results presented in this paper seem plausible, a number of future developments are described 
in this section which may lead to further refinements of the proposed WA subsystem.

A first point of departure for future work on the research conducted in this paper is to investigate the 
possibility of modelling the WA problem as a multi-objective, dynamic WA model. As mentioned in §3.2.4, 
one possibility is to formulate the WA problem as the variant of the VRPTW, but with multiple objectives. 
Appropriate solution methodologies for solving such a model should also be investigated. The development 
of such a model will lead to the completion of the proposed four classes of WA model in Figure 3.1. 

A further development may be to formulate other WA models within the four model classes depicted in 
Figure 3.1 to include in the WAM component as well as to investigate appropriate solution methodologies 
for each of these models, or alternative solution methodologies for the models proposed in this paper.

Finally, other SBAD scenarios may be simulated in terms of which the quality of solution of the collection 
of WA models may be ascertained.

10 Notes

1Scenario fulfilment occurs when operators work under stressful conditions, confusing a training scenario with 
reality and responding accordingly (Ebrahimpour, 2010). Operators typically ignore sensory information 
which contradicts the scenario. Roberts (1992) describes scenario fulfilment as “you see what you expect.”

2This includes changes in any of the entries in the TEWA database (e.g. sensor, target, WS or DA track 
changes).

3Typical MCDA methods which may be used within the TEFM component include the analytic hierarchy 
process of Thomas Saaty (Winston, 2004), goal orientated models such as goal programming and aspiration 
level models, or outranking models such as ELECTRE (Belton and Stewart, 2002).
The pitch-and-dive flight path attack technique, also known as the *combat hump dive* flight path attack technique, consists of an aerial threat approaching a DA at low altitude so as to attempt avoiding radar detection (Roux and Van Vuuren, 2008). This approach phase is followed by a manoeuvre phase during which the threat pulls up (pitches) at a distance of approximately 3–9km from the DA; small manoeuvres are possible during this stage. The threat then turns in to the DA so as to point directly at the DA. The threat now enters the attack phase during which the aircraft is stabilised before aiming at the DA. The weapons are released at a distance of approximately 800–2,500m from the DA. Once the weapons are released, the threat pulls up from the dive to manoeuvre away from the DA as quickly as possible. The pitch-and-dive flight path attack technique is illustrated graphically in Figure 10.1.

![Graphical illustration of the pitch-and-dive flight path attack technique](image)

Figure 10.1: Graphical illustration of the pitch-and-dive flight path attack technique (Roux and Van Vuuren, 2008).

The toss bomb flight path attack technique consists of an aerial threat approaching a DA at a low-altitude, approximately 60–150m above the ground (Roux and Van Vuuren, 2008). During the manoeuvre phase, the threat pulls up at an exact, pre-determined position (typically 4.5–6km from the DA) and starts aiming at the DA. It releases its weapons at a slant distance of approximately 3.5–4.7km from the DA. After releasing its weapons, the threats pulls around in order to manoeuvre away from the DA as quickly as possible. The toss bomb flight path attack technique is illustrated graphically in Figure 10.2.

A genetic algorithm allows for a pool of candidate solutions to evolve in a controlled environment over a number of iterations in an attempt to uncover near-optimum solutions in a process mimicking Darwinian natural selection.

In a tournament selection procedure a small subset of solutions, known as a *tour*, is chosen randomly, and the number of solutions in the subset is called the *tour size*. One solution is chosen from the tour to include as a parent solution in a so-called *mating pool* of solutions. The number of parent solutions included in the mating pool is called the *pool size*. Solutions are typically chosen according to their fitness function values as a selection criterion (i.e. the solution achieving the best objective function value is chosen as the parent solution).

Crossover is achieved by considering a random row $i$ of two parent solutions and then randomly selecting one of these rows to use in row $i$ of the child solution.

Mutation is achieved by randomly selecting a row in the parent solution and changing all the entries in the row to zero. A random column is then selected and the entry corresponding to the originally selected row is assigned a value of one.

The NSGA II aims to find a good approximation of the Pareto optimal solutions to the model.
The fast nondominated sorting algorithm works in such a way that the nondominated solutions are contained in the first front and are assigned a rank value of 1, followed by the solutions that are dominated once (those in the second front), which are assigned a rank value of 2, etc.

The selection procedure implemented in the NSGA II is similar to the tournament selection procedure described in §6.1, except for the selection criteria. In the NSGA II, parent solutions are selected based on two selection criteria. Firstly, the rank value of a solution is considered. When comparing two solutions, the solution achieving the lowest numbered rank value is considered more favorable and is selected as the parent solution. If two solutions have the same rank value, a second selection criterion, known as a *crowding distance density measure*, is used to select a parent solution. This measure quantifies the density of solutions in the objective space and such a value is calculated for each candidate solution. A high value indicates that a solution is more isolated, while a low value indicates that a solution is more crowded. A solution achieving a higher crowding distance value between two solutions with the same rank value is considered more favorable.

A fixed mean approach entails calculating the mean value of a subset of consecutive stages in an entire set of stages. The stages chosen for inclusion in the calculation should be consecutive and should also contain the stage achieving the smallest value in the entire set of stages. For the scenario described in §5, the fixed mean value for each FW was calculated by taking the mean survival values of the threats for four consecutive time stages in the entire FW period.

Simulated annealing is an optimisation solution methodology which mimics the annealing process involved in metallurgy. During this process, heat treatment is applied to a material in order to change its molecular properties. The material is typically heated above the recrystallization temperature, after which it is cooled down very slowly. The heating of the material causes the atoms to become excited and they start to vibrate randomly through higher energy states. The material is then cooled down slowly in order for the atoms to vibrate less until they settle down into a low energy state. This slow cooling process allows for a better chance of the atoms settling into a lower energy state than the initial energy state (Bertsimas and Tsitsiklis, 1993).

The neighbourhood move operator performs an intelligent move by first selecting a threat to assign to a WS and then by assigning it a feasible FW, taking into account the FWs of other threats which may be assigned to the same WS (Van der Merwe and Van Vuuren, 2013). If other assignments are affected by the move, they are adjusted in order for the solution to remain feasible.
A move that improves the objective function are always accepted, while a move that does not improve the objective function is accepted according to the classical Metropolis rule (Cai et al., 2010) i.e. with probability \( \exp(-\Delta_{obj}/t) > r \), where \( \Delta_{obj} \) denotes the change in the objective function when moving from the current solution to a randomly chosen neighbouring solution, \( t \) denotes the temperature which controls the randomness of the search and \( r \) denotes a random number between the values 0 and 1. If the neighbouring solution is accepted, it is taken as the new current solution, while if the neighbouring solution is rejected, the current solution remains unchanged. If the accepted neighbouring solution achieves a better objective function value than that of the incumbent solution, it is taken as the new incumbent solution.

A high starting temperature is typically implemented in order to ensure that most transitions are accepted during the initiation iterations of the algorithm. In contrast, a sufficiently low stopping temperature is required in order for the algorithm to converge towards a local minimum.

The cooling schedule indicates how and when the temperature should be lowered. Other cooling schedules that may be considered for implementation in the algorithm includes a geometric cooling schedule (Vigeh, 2011) and an adaptive cooling schedule (Huang et al. 1986).

The incumbent solution was taken as the best solution uncovered during the search rather than the last current solution, since the algorithm may accept worsening solutions during its execution. This may lead to a poor current solution when the algorithm terminates.

The reason for the priority weighted accumulated survival probability of the threats being lower than the results achieved by the single-objective, static WA model (presented in §6.1) and the multi-objective, static WA model (presented in §6.2) is due to the fact that the single-objective, dynamic WA model aims to minimise the priority weighted accumulated survival probability of the threat over the entire predicted time window. Hence, the model considers survival probabilities of threats over a number of FWs and minimises these values at once, rather than for a single stage.

### 11 References


Van Staden, H.E. 2012. Postgraduate student at Stellenbosch University. Personal Communication. Contactable at: heletjevs@gmail.com


Visser, B. 2008. Retired military expert. Personal Communication. Contactable at: bvisser@rrs.co.za