A unified methodology for single- and multiobjective in-core fuel management optimisation based on augmented Chebyshev scalarisation and a harmony search algorithm

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ABSTRACT

The in-core fuel management optimisation (ICFMO) problem is the problem of finding an optimal fuel reload configuration for a nuclear reactor core. ICFMO may involve the pursuit of a single or multiple objectives, while satisfying several constraints. Very little multiobjective ICFMO research involving the fundamental notion of Pareto optimality has, however, been performed. In this paper, a unified methodology is proposed for the modelling and solution of single- and multiobjective ICFMO problems, be they constrained or unconstrained. With this methodology, ICFMO problems incorporating a variety of objectives and/or constraints may be modelled and solved rapidly, thus providing a cycle-to-cycle optimisation decision support capability for nuclear reactors. An augmented Chebyshev scalarising objective function is incorporated in the methodology for modelling any number of objectives, while an additive penalty function handles potential constraints. Furthermore, an adapted harmony search algorithm is used to solve a given ICFMO problem. The algorithm is able to yield a single solution or a nondominated set of solutions as result (depending on the number of objectives in a problem). The applicability of the methodology is demonstrated by solving (approximately) a variety of ICFMO test problems for the SAFARI-1 nuclear research reactor. The results indicate that the methodology may be used as an effective decision support tool for reactor operators tasked with designing reload configurations from cycle to cycle.

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1. Introduction

The in-core fuel management optimisation (ICFMO) problem is the problem of finding an optimal fuel reload configuration (or loading pattern) for a nuclear reactor core. In order to find such an optimal configuration, fresh and partially-burnt fuel assemblies have to be assigned to loading positions in a reactor core so that the resulting configuration optimises reactor performance, while also ensuring that certain operational constraints are satisfied. There are a number of characteristics which make the ICFMO problem difficult to solve, such as a large disjoint feasible decision space, nonlinear objectives and constraints, and significant computational complexity (Turinsky, 2005).

ICFMO has been studied for several decades, with the majority of the research orientated towards power reactors and in the context of single-objective optimisation. Various solution techniques have been proposed for solving the ICFMO problem during this time. These techniques include mathematical programming methods, expert/knowledge-based systems and, more recently, metaheuristics such as simulated annealing, tabu search, evolutionary and swarm intelligence algorithms (Meneses et al., 2010). Metaheuristics are approximate solution techniques designed specifically for obtaining high-quality solutions to optimisation problems within reasonable computation times. Therefore, a solution produced by a metaheuristic may or may not be (globally or even locally) optimal and as such, it is deemed an approximate solution.

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to a problem. In contrast, an exact solution technique guarantees a globally optimal solution to a problem, given sufficient computation time.

In many instances, ICFMO involves multiple objectives and as such, the multiobjective ICFMO problem may be described as the problem of finding a Pareto optimal set (see Section 2) of fuel reload configurations for a nuclear reactor core. Very little multiobjective ICFMO research involving the fundamental notion of Pareto optimality has been performed. Works on the topic include those of Parks (1996), Hedayat et al. (2009) and Park et al. (2009). In most research papers, scalarising approaches involving linear weighted sum aggregations of the objectives are adopted, which yield a single solution (Gong et al., 2011; Liu and Cai, 2012; Pursalehi et al., 2013). As such, these approaches do not solve the multiobjective ICFMO problem in terms of finding a set of Pareto optimal solutions. Furthermore, scalarisation by means of the linear weighted sum approach has a serious shortcoming since it cannot uncover all the Pareto optimal solutions if the problem is nonconvex (Miettinen, 2008). In previous works by the first three authors of this paper, a linear weighted sum approach (Schlünz et al., 2012), as well as an augmented weighted Chebyshev approach (Schlünz et al., 2013) were adopted for the ICFMO problem. Although the latter approach yielded improved multiobjective results in a Pareto optimality sense, it was burdensome with respect to user-defined input parameters and had constraint limitations. That methodology also does not yield an approximate Pareto set as result.

In this paper, a unified methodology based on augmented Chebyshev scalarisation and a harmony search algorithm is proposed for the modelling and solution of single- or multiobjective ICFMO problems, be they constrained or unconstrained. With this methodology, ICFMO problems containing a variety of objectives and/or constraints may be modelled and solved rapidly, thus providing cycle-to-cycle optimisation capabilities for nuclear reactors. Such capabilities may be especially useful for research reactors responding to changing research or commercial demands with relatively short turnaround times, or new safety requirements, etc. The SAFARI-1 nuclear research reactor in South Africa is an example of such a reactor, where flexibility in the reload configuration so as to accommodate utilisation requirements is essential for the successful operation of the reactor.

The modelling approach within the methodology addresses the shortcomings of the weighting method, while also enabling the usage of the same solution technique for solving single- or multiobjective optimisation problems. Essentially, any capable single-objective solution technique can be applied to solve an ICFMO problem modelled according to the proposed modelling approach. A metaheuristic solution technique called harmony search (HS) can be used to solve a given ICFMO problem within the methodology. It is able to yield a high-quality solution or a high-quality nondominated set of solutions for single- or multiobjective optimisation problems, respectively. The applicability of the methodology is then demonstrated by solving single-objective and multiobjective ICFMO test problems for the SAFARI-1 reactor.

The paper is organised as follows. A brief introduction to single- and multiobjective optimisation is presented first. Then, the proposed modelling approach for ICFMO, consisting of the constraint handling technique and the scalarising objective function, is described. This is followed by an overview of the solution methodology that has been employed, as well as the specific adaptations made to it. The section thereafter contains a description of the SAFARI-1 reactor and the test problems that were considered within this context. The results obtained for these test problems are then presented, followed by the conclusions of the paper.

2. Optimisation definitions and notations

A model for an optimisation problem typically includes the following components: objective function(s), decision variables, and constraints (Winston, 2004). Objectives for an optimisation problem are chosen according to the goals of a decision maker and they constitute the specific aims a decision maker wants to achieve. Decision variables are those variables within an optimisation problem whose values are under the control of a decision maker. These variables (and their values) influence the performance of the system being modelled in terms of the objective(s) adopted. In many instances, however, only certain values (or combinations of values) of the decision variables are possible. These restrictions constitute the constraints of the optimisation problem.

A single-objective optimisation problem (SOP) consists of maximising/minimising one objective function over some decision space, while a multiobjective optimisation problem (MOP) consists of simultaneously maximising/minimising two or more objective functions over some decision space. Suppose, without loss of generality, that all objective functions are to be maximised. Then the general formulation of a constrained optimisation problem with \( q \) objective functions \( f_1(x), \ldots, f_q(x) \) may be written as

\[
\begin{align*}
\text{maximise} & \quad f(x) = \left[ f_1(x), f_2(x), \ldots, f_q(x) \right], \\
\text{subject to} & \quad g_i(x) \leq 0, \quad i = 1, \ldots, r, \\
& \quad h_j(x) = 0, \quad j = 1, \ldots, s, \\
& \quad x \in \mathbb{R}^n, \\
\end{align*}
\]  

where \( x = [x_1, \ldots, x_n] \) denotes the vector of decision variables \( \text{(hereafter referred to as the decision vector)} \), \( g_1(x), \ldots, g_r(x) \) are the inequality constraint functions and \( h_1(x), \ldots, h_s(x) \) are the equality constraint functions. A decision vector \( x \) satisfying all the constraints in (1) is called a feasible decision vector to (1). If \( r - s = 0 \) in (1), then the problem is called unconstrained; otherwise it is a constrained problem. Furthermore, if \( q = 1 \), then problem (1) is an SOP, while if \( q > 1 \) it is an MOP.

Although an SOP may have a single solution \( x \) (not necessarily unique) that maximises its objective function, there is typically no single solution that can simultaneously maximise all the objective functions in an SOP. Solving an SOP instead yields a set of compromise or trade-off solutions, which leads to the notion of Pareto optimality. Let \( \mathcal{X} \) be the feasible domain specified by the constraints in (1), i.e. the set of all feasible decision vectors to (1).

**Definition 1.** A feasible decision vector \( x \in \mathcal{X} \) dominates another feasible decision vector \( x' \in \mathcal{X} \) if \( f_i(x) \geq f_i(x') \) for all \( i = 1, \ldots, q \) and \( f_j(x) > f_j(x') \) for at least one \( j \in \{1, \ldots, q\} \).

**Definition 2.** A feasible decision vector \( x \) in some subset \( \mathcal{S} \subseteq \mathcal{X} \) is nondominated in \( \mathcal{S} \) if there exists no feasible decision vector \( x' \in \mathcal{S} \) which dominates \( x \).

**Definition 3.** A feasible decision vector \( x' \in \mathcal{X} \) is said to be Pareto optimal if it is nondominated in \( \mathcal{X} \).

**Definition 4.** The Pareto set of (1), denoted by \( \mathcal{P}_\mathcal{X} \), is the set containing all the Pareto optimal feasible decision vectors to (1), while the Pareto front of (1), denoted by \( \mathcal{P}_F \), is the set containing all the objective function vectors corresponding to its Pareto set, i.e. \( \mathcal{P}_F = \{ f(x) | x \in \mathcal{P}_\mathcal{X} \} \).
3. The proposed modelling approach for ICFMO

As mentioned in Section 1, the optimisation methodology proposed here unifies SOP and MOP variants of the ICFMO problem, which may be constrained or unconstrained. It is explained in this section how this is achieved. An advantage of the unified methodology is the fact that SOPs and MOPs may be modelled in the exact same manner, and the same single-objective solution technique may be employed for solving either variant of the problem.

3.1. The problem statement

The methodology covers ICFMO problems within the following scope. Objectives and constraints should be expressible as parameters that may be obtained from a reactor core calculation system. Any reload configuration should comprise an assignment of fuel assemblies (from some known set of fuel assemblies) to fuel loading positions in a reactor core, as illustrated in Fig. 1. As stated in Section 1, the methodology is intended for solving ICFMO problems with various objectives and/or constraints, on a cycle-to-cycle basis. Therefore, a solution yielded by the methodology is valid only for the coming operational cycle, and is not necessarily a long-term (multi-cycle) solution.

Let $n$ be the number of fuel loading positions, labelled $1, \ldots, n$, in a reactor core, and let $m$ be the number of available fuel assemblies, labelled $1, \ldots, m$, with $m \geq n$. A reload configuration may then be represented by a partial permutation decision vector $x = [x_1, \ldots, x_n]$ where $x_i = j$ denotes that fuel assembly $j \in \{1, \ldots, m\}$ is placed into fuel loading position $i \in \{1, \ldots, n\}$. This representation is also depicted in Fig. 1. Let $\mathcal{Y}$ be the set of all possible fuel reload configurations (i.e. partial permutation decision vectors). The general constrained ICFMO problem may then be formulated as

$$\begin{align*}
\text{maximise} \quad & f(x) = [f_1(x), \ldots, f_q(x)], \\
\text{subject to} \quad & g_i(x) \leq g_i^\text{lim}, \quad i = 1, \ldots, r, \\
& h_j(x) = h_j^\text{lim}, \quad j = 1, \ldots, s, \\
& x \in \mathcal{Y},
\end{align*}$$

(2)

where $g_i^\text{lim}$ and $h_j^\text{lim}$ are the (non-zero) limiting values for each of the constraint functions and the other symbols have the same meanings as before. Note that (2) is, therefore, a special case of (1). The vector $f(x)$ that contains the objective function values corresponding to a decision vector $x \in \mathcal{Y}$ is referred to as the objective vector.

3.2. The constraint handling technique

When solving a constrained optimisation problem, some technique is required for taking the constraints into account. One such technique is to disregard all decision vectors that violate any constraint, while another is to consider those decision vectors, but to penalise them. The latter approach is adopted in our proposed modelling approach. As such, an additive penalty function is employed as the constraint handling technique. According to this technique, if a decision vector violates any of the constraints, a corresponding penalty value related to the magnitude of that violation is incurred. The total penalty value of all the constraint violations is then subtracted (since we are solving a maximisation problem) from each of the $q$ objective function values in order to penalise the vector of decision variables for being infeasible. Likewise, the total penalty value would be added when solving a minimisation problem. A feasible decision vector, therefore, has a zero penalty.

Let

$$G(x) = \sum_{i=1}^r \max \left\{0, \frac{g_i(x) - g_i^\text{lim}}{g_i^\text{lim}} \right\}$$

be the total scaled constraint violation associated with the inequality constraints. Similarly, let

$$H(x) = \sum_{j=1}^s \frac{|h_j(x) - h_j^\text{lim}|}{h_j^\text{lim}}$$

be the total scaled constraint violation associated with the equality constraints.

Each constraint violation above has been scaled in order to create dimensionless values that are similarly scaled by order of magnitude. This allows for the various constraint violations to be combined in a responsible manner. Furthermore, all constraint violations are considered to be equally important, thereby eliminating the need for additional importance weights to be specified for different constraints.

The penalty function adopted in our proposed modelling approach for an ICFMO problem is then given by

$$P(x) = \gamma(G(x) + H(x)).$$

(3)

and is to be used in conjunction with a scalarising objective function, as described in the following section. By definition, $P(x) = 0$ for an unconstrained ICFMO problem.

The parameter $\gamma$ in (3) is a strictly positive severity factor, typically determined empirically. The value of $\gamma$ may be chosen such that the $q$ (penalised) objective function values of an infeasible decision vector are worse than the $q$ (unpenalised) objective function values of the majority of feasible decision vectors. By repeatedly solving the constrained ICFMO problem off-line for different values of $\gamma$, and evaluating the solution qualities obtained, an acceptable value may be settled upon and selected for use during the actual optimisation.

3.3. The scalarising objective function

In an SOP, there is only one objective and its corresponding objective function may therefore be optimised directly to yield a single solution (not necessarily unique). An MOP, however, has several objectives to be considered simultaneously and should yield a set of trade-off or Pareto optimal solutions, as explained in Section 2. One of the approaches available for multiobjective optimisation is to scalarise the multiple objectives into a single function to be optimised. Many of these scalarising approaches are theoretically grounded in order to ensure Pareto optimality (Miettinen, 1999).

In this paper, a scalarising objective function is utilised in the modelling approach for SOP and MOP variants of the ICFMO problem. The proposed function is based on augmented Chebyshev scalarisation (Miettinen, 1999; Stewart, 2007) and the inclusion of so-called aspiration levels. According to the modelling approach, the decision maker has to specify an aspiration level $z_k$ for each objective $k \in \{1, \ldots, q\}$ before optimisation commences. These reasonable or desirable aspiration levels form a so-called aspiration vector (or point), denoted by $x = [z_1, \ldots, z_q]$.

An aspiration level is defined here as an objective function value that would satisfy the decision maker if that value were to be achieved for the objective. Known target/goal values (based on historical data or expert judgement, etc.) may be taken as the aspiration levels, implying that objectives need only be improved up to sufficient satisfaction, and not necessarily to optimality. Another approach is to take unattainable, but realistic, values as the aspiration levels so as to improve the objectives as best possible, and not only up to the target values. Ideally, however, if individual SOPs
can be solved for each of the objectives in the MOP, then those optimised objective values should be taken as the aspiration levels, forming a so-called ideal objective vector.

The scalarising function, based on augmented Chebyshev scalarisation with aspiration levels, is defined here as

$$ z_q(x) = \max_{k \in \{1, \ldots, q\}} \left\{ \frac{f_k(x) - \alpha_k}{\alpha_k} \right\} + \lambda \sum_{l=1}^{q} \left\{ \frac{f_l(x) - \alpha_l}{\alpha_l} \right\} , $$

where $\lambda$ is a sufficiently small (positive) parameter ensuring that the augmentation term does not negate the Chebyshev term. Typical values for $\lambda$ that have been adopted in the literature include $\lambda = 10^{-3}$ (Miettinen and Mäkelä, 2002) and $\lambda = 10^{-6}$ (Romero et al., 1998).

The idea behind this approach is to minimise the Chebyshev distance between the objective vector $f(x)$ and the aspiration vector $\alpha$, as seen in the Chebyshev term in (4). Therefore, optimisation using this approach improves, at any given time, the worst deviation between any objective and its aspiration level. An advantage of using the Chebyshev norm is that resulting solutions are typically “well-balanced” with respect to all the objective function values (Stewart, 2007). If, for some specific decision vector, the worst deviation cannot be improved upon anymore, it may still be possible to improve achievements in the other objectives. Inclusion of the augmentation term in (4) allows this approach to achieve those possible improvements.

Most importantly, (4) has the property that its exact solution is Pareto optimal (Miettinen, 1999) for an appropriate choice of $\alpha$ (e.g. the ideal objective vector). As such, it addresses the shortcomings of the popular weighting method employed in the ICFCMO literature thus far. The resulting solution may not necessarily be to the satisfaction of the decision maker, hence it would be valuable if several (ideally Pareto optimal) solutions are found. Techniques exist whereby the scalarising function can be varied in order to generate a spread of Pareto optimal solutions (Miettinen, 2008). Unfortunately, these techniques are computationally expensive, because they typically require resolving the optimisation problem several times. Non-scalarising techniques can also yield nondominated solutions which approximate the Pareto set, but they typically require specialised multiobjective implementations which may lead to a loss of generality for solving single-objective optimisation problems as well.

A midway between these techniques is adopted in our proposed approach, where an archive of nondominated decision vectors is maintained during the solution of (4), using the same metaheuristic solution technique for single- and multiobjective ICFCMO problems. Nondominated decision vectors in the vicinity of the final approximate solution (possibly Pareto optimal) may be uncovered during the solution process. As such, it is possible to construct a nondominated set of solutions (called an approximate Pareto set) for an MOP by solving an SOP using (4). Such an approximate Pareto set will not necessarily exhibit a good spread over the true Pareto optimal set due to the fixed scalarising function, the choice of $\alpha$ and the metaheuristic solution technique adopted, but it will provide the decision maker with good alternative trade-off solutions to consider.

The scalarising function (4) assumes an unconstrained optimisation problem. The penalty function (3) should be added to it in order to accommodate constraints as well. Notice that the value of function (4) is similarly scaled by order of magnitude to the scaled constraint violations in the penalty function. Now, the scalarising objective function (to be minimised) for ICFCMO becomes

$$ z_q(x) = \max_{k \in \{1, \ldots, q\}} \left\{ \frac{f_k(x) - \alpha_k}{\alpha_k} \right\} + \lambda \sum_{l=1}^{q} \left\{ \frac{f_l(x) - \alpha_l}{\alpha_l} \right\} + P(x). $$

Fig. 1. Assignment of fuel assemblies to loading positions.
This scalarising function is not only applicable to the MOP variant of the ICFMO problem, but also to its SOP variant. For \( q = 1 \), the maximisation operator reduces to the identity-operator, while the augmentation term simply has the effect of scaling the function value. As such, the augmentation term has no influence on the optimisation process. The penalty function is already independent of the number of objectives. Therefore, if the aspiration level is chosen as an unattainable value and an exact solution technique is used, then minimising (5) for \( q = 1 \) will yield an optimal solution.

Now, the general ICFMO problem may always be formulated as a minimisation SOP within the proposed methodology, and is given by

\[
\text{minimise } z_q(x); \\
\text{subject to } x \in Y;
\]

where \( z_q(x) \) is defined in (5).

4. The proposed solution approach for ICFMO

As Turinsky (2005) mentions, the ICFMO problem is a high-dimensional optimisation problem containing nonlinear objectives and constraints that lack derivative information. A reactor core calculation system (i.e. a core simulator) is required for the neutronic evaluation of reload configurations. Reactor core parameter values, which may appear as ICFMO objectives or constraints, are determined from this neutronic evaluation. Unfortunately, such an evaluation is computationally expensive, resulting in limited time budgets for ICFMO.

Metaheuristic techniques are well-suited for optimisation problems exhibiting the above-mentioned properties, and have successfully been applied to ICFMO (Meneses et al., 2010). Recently, a metaheuristic called harmony search (HS) (Geem et al., 2001) has also been applied to ICFMO (Schlünz et al., 2012; Poursalehi et al., 2013b) and was found to yield competitive results when compared to a Hopfield neural network/simulated annealing hybrid method and a genetic algorithm. In this work, an adapted HS algorithm is used to solve problem (6) within the proposed methodology. This adaptation, however, is an alternative to the algorithm of Poursalehi et al. (2013b), and builds upon the algorithm adopted by Schlünz et al. (2013) by including nondominated decision vector archiving capabilities.

4.1. Harmony search

The HS algorithm is inspired by the musical process of seeking a perfect harmony (Geem et al., 2001). Analogous to a musical ensemble producing different sounds on their instruments in order to find a pleasing combination of sounds (i.e. harmonies), the HS algorithm combines different values of the decision variables to find an optimal solution. The basic HS algorithm consists of four steps:

Step 1. Initialise a memory structure, called the harmony memory (HM), with random\(^2\) harmonies (i.e. decision vectors).

Step 2. Improvise a new harmony according to guidelines that probabilistically consider the HM, local perturbations and randomisation.

Step 3. Compare the new harmony with the worst harmony contained in the HM. If the new harmony is better than the worst harmony, replace that worst harmony in the HM with the new harmony.

Step 4. If some specified termination criteria are met, terminate the process; otherwise, return to Step 2.

The guidelines on how to improvise a new harmony, referred to in Step 2, are described by an example. Consider a simple SOP with the goal of maximising an objective function \( f(x) \), subject to \( x_i \in X_i \) for \( i = 1, \ldots, \ell \), where \( x = [x_1, \ldots, x_\ell] \) is a vector of \( \ell \) continuous decision variables, and \( X_i = [0, 1] \) is the set of allowable values for variable \( i \).

Suppose \( \ell = 4 \) and that the HM (of size 3) at the start of some iteration during the algorithm’s execution is given by

<table>
<thead>
<tr>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( x_4 )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>0.44</td>
<td>0.05</td>
<td>0.23</td>
<td>15.6</td>
</tr>
<tr>
<td>0.50</td>
<td>0.25</td>
<td>0.99</td>
<td>0.75</td>
<td>10.0</td>
</tr>
<tr>
<td>0.82</td>
<td>0.60</td>
<td>0.31</td>
<td>0.79</td>
<td>18.2</td>
</tr>
</tbody>
</table>

A new decision vector \( x^* \) is created on a variable-by-variable basis. For each variable, a value is randomly selected, either from the HM, or from its allowable range. A parameter called the harmony memory consideration rate (HMCR), denoted by \( p_{\text{hmcr}} \), is introduced in order to guide the selection towards values from the HM. Consider the variable \( x_3 \) as an example: with probability \( p_{\text{hmcr}} \), a value is randomly selected from the list \([0.44, 0.25, 0.60] \). This list corresponds to the values in the shaded column \( x_3 \) of the HM, and may contain repeated values. Since each element in the list has an equal selection probability, repeated values have a higher probability of being selected. Alternatively, with probability \( 1 - p_{\text{hmcr}} \), a value is randomly selected from \( X_3 \). Typical values for the HMCR range between 0.7 and 0.95 (Yang, 2009).

In the event of a variable taking a value from the HM, a secondary process, called pitch adjustment, may be performed. This process perturbs the current value of the variable in order to create a slightly different decision vector. A parameter called the pitch adjusting rate (PAR), denoted by \( p_{\text{par}} \), determines the probability of performing the pitch adjustment. The size of the perturbation is determined by a bandwidth parameter, denoted by \( b_a \). So, with probability \( p_{\text{par}} \), the value of a variable may be adjusted according to

\[
x^*_{\text{new}} = x^*_{\text{old}} + b_a \cdot \varepsilon,
\]

where \( \varepsilon \) is a random number in the range \([-1, 1] \). Typical values for the PAR range between 0.1 and 0.5, while the value of \( b_a \) is problem-dependent (Yang, 2009).

The HM size is also problem-dependent and may be determined empirically. If the HM size is too small, the HS algorithm may converge prematurely while if the HM size is too large, the algorithm may reduce to a pure random search.

4.2. The adapted harmony search algorithm

The HS algorithm has to be adapted for use in the context of ICFMO. These adaptations are twofold. First, decision vectors to problem (6) are represented as partial permutations and, therefore, Step 2 of the HS algorithm should explicitly preserve the structure of partial permutations, and do so in an unbiased manner. Secondly, feasible nondominated decision vectors have to be archived during optimisation if \( q \geq 2 \) in (6) so that an approximate Pareto set may be returned upon algorithm termination. The adapted HS algorithm is presented in pseudocode form as Algorithm 1.

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\(^2\) Where any reference is made to a random selection in this paper, the selection is assumed to be performed according to a uniform distribution.
The allowable lists that are constructed in line 8 of Algorithm 1 comprise the following. The list $M$ contains those fuel assembly labels in the HM corresponding to the selected position $i$, excluding those labels already selected in other positions of $x_0$. Similarly, the list $R$ contains all the fuel assembly labels, excluding those labels already selected in other positions of $x_0$. A simple example of how these lists are constructed is given next.

Suppose $n = 4$ and $m = 5$ and that the HM (of size 4) at the start of some iteration of the algorithm is given by $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 4 & 3 & 2 \\ 3 & 1 & 5 & 4 & 1 \end{pmatrix}$. While a new decision vector has only been generated partially as $x_0 = \{5, x_2, x_3, 3\}$, suppose the new loading position is randomly selected as $i = 3$ (line 7 of Algorithm 1). Then, $M = \{2, 4, 5\} \setminus \{5, 3\} = \{2, 4\}$, corresponding to the labels in the shaded column $x_3$ of the HM, excluding the labels already selected in the partial solution $x_0$. Similarly, $R = \{1, 2, 3, 4, 5\} \setminus \{5, 3\} = \{1, 2, 4\}$.

The pitch adjustment process has been adapted significantly from the original HS algorithm, and is presented in pseudocode form as Algorithm 2. Pitch adjustment has been redefined to perform pair-wise exchanges between variables in the fully generated decision vector. Therefore, pitch adjustment is not performed per variable during the creation of a new decision vector, but is rather performed after the decision vector has been fully generated.

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**Algorithm 1: HS algorithm for the integrated ICFMO problem**

1. Initialise HM (of some chosen size) with randomly generated decision vectors, each corresponding to a random partial permutation of $m$ unique fuel assembly labels, chosen from $\{1, \ldots, m\}$.
2. Initialise the feasible nondominated decision vector archive $A$ with the nondominated decision vectors in HM.
3. while algorithm stopping criteria not met do
   4.   Reset the new decision vector $x'$
   5.   Reset the pitch adjusting set $P \leftarrow \emptyset$
   6.   while new decision vector only partially created do
      7.     Randomly select a new loading position $i$
      8.     Construct the allowable memory consideration list $M$, and the allowable random list $R$
      9.     Generate a random number $r \sim U(0, 1)$
     10.    if $r \leq p_a$ then
         11.       Randomly select a fuel assembly label for position $x_i'$ from $M$ if $M \neq \emptyset$;
            otherwise, select it from $R$
         12.       $P \leftarrow P \cup \{\text{loading position } i\}$
     13.    else
         14.       Randomly select a fuel assembly label for position $x_i'$ from $R$
     15.    end
     16. end
     17. Perform the pitch adjustment process using Algorithm 2
     18. Evaluate new decision vector $x'$ with a core simulator and determine $z_q(x')$
     19. if $z_q(x') < \text{maximum } z_q \text{ in HM}$ then
         20.           Replace decision vector in HM having the maximum $z_q$ with new decision vector $x'$
     21. end
     22. $A \leftarrow \text{feasible nondominated decision vectors in } A \cup \{x'\}$
4. end

**Algorithm 2: Pitch adjustment procedure**

**Input**: decision vector $x'$ and pitch adjusting index list $P$  
**Output**: Pitch adjusted decision vector $x''$

1. while $|P| \geq 2$ do
   2.   Randomly select loading position $i \in P$
   3.   Generate a random number $r \sim U(0, 1)$
   4.   if $r \leq p_a$ then
      5.     Find $k_+$ such that $x_{i_+}' = x_+ + 1$
      6.     Find $k_-$ such that $x_{i_-}' = x_+ - 1$
      7.     if $k_+$ or $k_-$ found then
         8.           Randomly select loading position $k \in \{k_+, k_-\}$
         9.           Exchange $x_i'$ and $x_k'$
      10.   end
   11. end
   12. if $j \in P$ then
      13.     $P \leftarrow P \setminus \{i, k\}$
      14. else
      15.     $P \leftarrow P \setminus \{i\}$
16. end
17. end

Those variables that obtained values from memory consideration are stored in the pitch adjusting list $P$. The cardinality of the list is denoted by $|P|$. During pitch adjustment, these variables may then only be pair-wise exchanged with variables that have neighbouring values, as seen in lines 5 and 6 of Algorithm 2.

The archive containing the nondominated feasible decision vectors is updated at the end of each iteration in order to consider the
newly created decision vector for possible inclusion. If there is only one objective in the ICFMO problem, then this archive simply contains the best feasible decision vector found thus far.

As mentioned in Section 1, metaheuristics are approximate solution techniques and do not necessarily guarantee optimality. Therefore, the terms “approximate solution” and “approximate Pareto set” are adopted in this paper when referring to results yielded by the HS algorithm.

5. The test problems

The unified optimisation methodology proposed in this paper has been used to model and solve several ICFMO test problems in order to demonstrate its applicability. The SAFARI-1 nuclear research reactor is considered here as a case study, along with the reactor core calculation system, OSCAR-4.

5.1. The SAFARI-1 research reactor

SAFARI-1 is a 20 MW tank-in-pool type materials testing reactor in South Africa. The reactor is utilised for nuclear and materials research, as well as commercial irradiation services. Neutron scattering, radiography and diffraction experiments are performed at the reactor, utilising a number of neutron beam tubes located around the core. Several in-core irradiation positions are utilised for isotope production (primarily molybdenum-99 (99Mo) isotopes) while an ex-core facility is utilised for silicon transmutation doping.

The OSCAR (Overall System for the Calculation of Reactors) code system is used as the primary calculation tool to support the day-to-day operation planning, core-follow and reload simulation of the SAFARI-1 reactor (Stander et al., 2008). In order to evaluate reload configurations during optimisation, the three-dimensional multigroup nodal-diffusion simulator within the OSCAR-4 system is employed for core calculations. A top view of the core layout of the three-dimensional SAFARI-1 model used in the OSCAR-4 system is presented in Fig. 2.

The SAFARI-1 core consists of a 9 x 8 lattice which houses twenty-six low-enriched fuel assemblies, six control rods of fuel-follower type, seven dedicated 99Mo production rig facilities, two general isotope production rig (IPR) facilities, as well as other core components which we do not specify in detail. In the test problems to follow, only twenty-six fuel assemblies are considered for reload configuration optimisation, thus \( m = n = 26 \) in (2). The locations of the beam tubes and the silicon doping facility are indicated in Fig. 2. Note, however, that these locations are considered for identification and tally purposes only — the tubes and facility are not explicitly modelled in the OSCAR-4 SAFARI-1 model.

A typical shutdown and reload period for SAFARI-1 lasts five days. Therefore, an acceptable computational run time for performing ICFMO should not exceed three days. Given that OSCAR-4 requires approximately four minutes to evaluate the quality of a single fuel reload configuration on a personal computer with a 3.30 GHz CPU and 4 GB of RAM, three days of computation time correspond to approximately 1000 evaluations of the objective function \( z_q(x) \) in (5).

SAFARI-1 is currently operated according to a reload configuration design philosophy based on the well-known highest-mass to lowest-flux (HMLF) strategy. This strategy attempts to flatten the power profile over the reactor core and is well-suited to the needs of SAFARI-1. It results in a safe configuration for an operational cycle while generally meeting the utilisation requirements of the reactor, especially in terms of molybdenum production.

An actual operational cycle for SAFARI-1 during the year 2012 is considered in this paper for benchmarking purposes. The reload configuration loaded into the core during that cycle will hereafter be referred to as the historical solution. It was designed according to the philosophy based on HMLF and will be treated as a basis for comparison in the test problems to follow. The test problems considered in this paper consist of hypothetical scenarios in which the operators at SAFARI-1 have to deviate from their current design philosophy for the current operational cycle because new safety and utilisation requirements are supposedly imposed on the reactor. As such, the fuel assemblies used in the test problems correspond to those loaded into the core in the historical solution.

5.2. The test problems

Three different test problems for SAFARI-1 are considered in this paper in order to demonstrate the applicability of the method-
ology to diverse cycle-specific ICFMO problems. A new set of safety and utilisation requirements is imposed on all three problems, resulting in the problems being potentially more difficult to solve. The first problem is an SOP variant of the ICFMO problem, while the second and third problems are MOP variants. In general, the objectives associated with ICFMO problems are translated into parameters that may be returned by a core simulator. These translations are also described for the problems.

The newly imposed safety and utilisation limits for SAFARI-1 reload configurations, hereafter referred to as the standard constraint set, consist of the following inequality constraints:

- **Constraint 1.** The total $^{99}$Mo production must be greater than the demand.
- **Constraint 2.** The $^{99}$Mo yield for each molybdenum rig must be above a minimum limit.
- **Constraint 3.** The maximum axial thermal neutron flux level for each IPR must be above some threshold.
- **Constraint 4.** The relative and absolute power peaking factors must be below safety limits.
- **Constraint 5.** The total control bank worth must be above the safety limit.
- **Constraint 6.** The shutdown margin must be above the safety limit.

The specific limiting values of these constraints are proprietary knowledge and are therefore not divulged in this paper. The historical solution no longer meets all of the new requirements, specifically that of Constraint 3 which is violated by approximately 9%.

In the first problem scenario, SAFARI-1 is required to operate during the current cycle beyond its typical length for an extended period of time. This corresponds to one of the popular objectives for ICFMO problems in the literature, namely to maximise the fuel cycle length of a loaded core (i.e. the lifetime for which the reload configuration will remain critical) (Hedayat et al., 2009; Shaukat et al., 2010). This objective may be translated to maximising the effective multiplication factor, or maximising the excess reactivity of the core. Therefore, the first test problem to be considered, hereafter referred to as Problem 1, is the SOP of maximising excess reactivity, subject to the standard constraint set.

In the second problem scenario, SAFARI-1 is required to enhance its beam line research utilisation for the current cycle. In the OSCAR-4 diffusion code, beam line structures are not explicitly modelled and this objective may therefore be translated to maximising the average thermal neutron flux over the beam faces. As such, the second test problem, hereafter referred to as Problem 2, is the MOP of simultaneously maximising the specified neutron flux in beam tubes 1 + 2 and in beam tube 5 (see Fig. 2), subject to the standard constraint set.

The commercial services rendered by SAFARI-1 may be subdivided into the production of $^{99}$Mo isotopes, the production of other isotopes, and silicon doping. In the third problem scenario, SAFARI-1 has to respond to temporary market conditions and optimise its commercial services during the current cycle. The $^{99}$Mo production objective may be translated to maximising the assembly-averaged power levels in all the molybdenum rigs (see Fig. 2). Similarly, the production of other isotopes objective may be translated to maximising the maximum axial thermal neutron flux level in each of the IPRs (see Fig. 2). The silicon doping objective may also be translated to maximising the average thermal neutron flux level over the silicon doping facility (see Fig. 2). Therefore, the third and last test problem, hereafter referred to as Problem 3, is the MOP of simultaneously maximising the specified power levels in the molybdenum rigs, maximising the specified flux levels in each of the IPRs, and maximising the specified flux level in the silicon facility, subject to the standard constraint set.

In order to avoid potential confusion in Problem 3, note that the same quantity or parameter is allowed to appear as both an objective and a constraint. For example, even though molybdenum production is to be maximised, it is also constrained from below. Since improving in one objective typically occurs at the cost of deteriorating in another objective, such a constraint ensures that the molybdenum objective does not deteriorate beyond a threshold. Likewise for the specified flux levels in the IPRs.

In accordance with the proposed methodology, a decision maker has to specify aspiration levels for each of the objectives in the three problems above. In this study, an unattainable aspiration level has been chosen for the excess reactivity objective in Problem 1 so that an optimal solution may be sought (as explained in Section 3.3). Realistic aspiration levels, based on previous SOP results and expert judgement, have been chosen for all the objectives in Problems 2 and 3. These aspiration levels are also proprietary knowledge and are therefore not disclosed in this paper.

6. Results

A personal computer (Intel® Core™ i5-2500 CPU @ 3.30 GHz, 4 GB RAM, 32-bit operating system) was utilised for the calculations in this study. Algorithm 1 was used to solve each test problem multiple times, independently on the same computer, resulting in ten algorithmic executions per problem. Each of these executions used a different random seed, since HS is a stochastic algorithm which may produce different results each time it is executed. As such, descriptive statistics could be gathered for an analysis of the optimisation results. In practice, the best overall result or a combination of some results may be used for decision making purposes.

A brief empirical study was performed in order to determine reasonable values for the free parameters of Algorithm 1. They were chosen as: $HMSize = 15$, $p_c = 0.95$ and $p_m = 0.25$. As motivated in Section 5.1, the algorithm’s stopping criterion was set to a maximum of 1,000 iterations. Furthermore, the augmentation parameter in function (5) was chosen as $\lambda = 0.01$ and the constraint severity factor as $\gamma = 2.5$.

The optimisation results thus obtained were then compared to the historical solution. By comparing the results in this manner, it is possible to demonstrate the potential and applicability of the unified methodology to the decision makers at SAFARI-1 when faced with new demands for a coming cycle. Not only does the comparison illustrate the improved performance(s) in the required objective(s), it also demonstrates that the newly imposed standard constraint set is automatically satisfied during optimisation (i.e. feasible solutions are produced).

Since the optimal solution to Problem 1 and the Pareto optimal solutions to Problems 2 and 3 are unknown, the accuracy of the proposed methodology cannot be determined explicitly. Instead, quasi-reference solutions have been determined for the problems. These solutions are used for comparative purposes in order to gauge whether the accuracy of the methodology is acceptable. The procedures adopted for obtaining these quasi-reference solutions are as follows.

Problem 1 is modelled within a classical single-objective context (e.g. maximise $f(x)$, subject to $x \in X$) and not within the proposed approach. An aspiration level, therefore, cannot impede the search for optimality. Since HS has already been demonstrated in the ICFMO literature as an effective solution technique (Schlünz et al., 2012; Poursalehi et al., 2013a), it is used to solve Problem 1 in its classical single-objective context. However, twice as many iterations than before are used during the solution process, and a quasi-reference solution is thus obtained. Similarly, Problems 2 and 3 are modelled within a classical multiobjective context.
(e.g. maximise \( f_1(x), f_2(x), \ldots, f_q(x) \), subject to \( x \in X \)) and not within the proposed approach. Then, a fully-fledged multiobjective metaheuristic is used to solve Problems 2 and 3, also using twice as many iterations than before, and quasi-reference Pareto sets are thus obtained. The multiobjective optimisation using cross-entropy method (MOO CEM) metaheuristic (Bekker and Aldrich, 2011) has been adopted for this purpose, and its effectiveness has also been demonstrated in the ICFMO literature (Schlünz et al., 2014).

6.1. Problem 1: Fuel cycle length

The best feasible solution obtained across the ten independent algorithmic executions for Problem 1 yielded an improvement of 29.5% in the excess reactivity over that of the historical solution. Furthermore, the average over the ten executions for the best improvement was 28.5% with an absolute standard deviation of 0.4%. In Fig. 3, the convergence graph of the average best-found excess reactivity over the ten executions is presented, along with the standard deviation band. In addition, the average best-found excess reactivity over multiple executions for the solution of Problem 1 in the classical context is also presented. Note that the values have been scaled according to the percentage improvement in excess reactivity over that of the historical solution.

It is observed that the algorithm sharply improves the excess reactivity within the first 250 iterations, followed by a more gradual improvement up to the final iteration. Furthermore, the narrow band of the standard deviation in the latter half of the iterations confirms that the different algorithmic executions yield results of similar quality. It is also observed that the convergence of the proposed methodology is very similar to that of the classical approach. The quasi-reference solution yielded an improvement of 29.9% in the excess reactivity — only 0.4% better than the best-found solution. The proposed methodology is therefore capable
of yielding high-quality solutions within a limited computational budget for an SOP.

In Fig. 4, the reload configurations of the historical solution and the best-found solution for Problem 1 are presented visually in terms of the uranium-235 mass in each fuel assembly. It is observed that the best-found configuration mimics a type of in-out loading, consistent with what we would expect from a core which maximises the fuel cycle length. In contrast, the historical configuration is almost the opposite thereof and mimics more of an out-in loading. As such, the results of this test problem clearly show that the methodology is able to propose a good approximate solution for a problem which has solutions that significantly depart from the current SAFARI-1 reload philosophy.

6.2. Problem 2: Research utilisation

The final feasible, nondominated fronts obtained by the ten algorithmic executions for Problem 2 are presented together in Fig. 5, along with the historical solution. The values have again been scaled according to the percentage improvement in each objective over that of the historical solution. An approximate Pareto front was determined from these results by performing a nondominated sorting over the combined fronts. This approximate Pareto front is also presented in Fig. 5, along with a quasi-reference Pareto front obtained by solving Problem 2 using the MOO CEM algorithm with twice as many iterations.

It is observed that the approximate Pareto front yields improvements between 2.5% and 6% in the performance of beam tubes 1 + 2 over that of the historical solution, at the cost of decreases between 3% and 12% in the performance of beam tube 5. As such, the methodology is able to yield a good set of trade-off solutions which meets all the newly imposed constraints, allowing for flexibility in decision making by a SAFARI-1 operator. It is further observed that the approximate front obtained using the methodology approaches the quasi-reference Pareto front. The methodology, therefore, succeeds in suggesting feasible solutions that improve upon the historical solution at a trade-off cost, and yields an approximate Pareto front of acceptable accuracy for an MOP within a limited computational budget.

In order to demonstrate the effect of Constraint 3 on the solution quality, the approximate Pareto front above is compared to a modified front obtained from the same results, but now considering decision vectors which also violate Constraint 3 up to the same level as the historical solution. These fronts are presented in Fig. 6. It is observed that much better performance is possible when Constraint 3 has a lower limiting value. It is expected that the optimisation of Problem 2 using such a lower constraint limit would yield even better results than those of the modified front in Fig. 6.

6.3. Problem 3: Commercial services

An approximate Pareto front was determined from the results of all ten algorithmic executions for Problem 3 in the same manner as before. This front is not represented visually since the results are four-dimensional. Instead, a so-called payoff table is employed in order to present an approximation of the available ranges found in each objective. The payoff table for Problem 3 contains values for the approximate Pareto front, as well as for the quasi-reference Pareto front obtained using the methodology, while the values in brackets correspond to the quasi-reference Pareto front.

Table 1
Payoff table for Problem 3.

<table>
<thead>
<tr>
<th>Objective with best performance</th>
<th>Percentage improvement (%) obtained for:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>99Mo</td>
</tr>
<tr>
<td>10%</td>
<td>0.7%</td>
</tr>
<tr>
<td>20%</td>
<td>1.8%</td>
</tr>
<tr>
<td>30%</td>
<td>2.7%</td>
</tr>
<tr>
<td>40%</td>
<td>3.7%</td>
</tr>
<tr>
<td>50%</td>
<td>4.7%</td>
</tr>
<tr>
<td>60%</td>
<td>5.7%</td>
</tr>
<tr>
<td>70%</td>
<td>6.7%</td>
</tr>
<tr>
<td>80%</td>
<td>7.7%</td>
</tr>
<tr>
<td>90%</td>
<td>8.7%</td>
</tr>
<tr>
<td>100%</td>
<td>9.8%</td>
</tr>
</tbody>
</table>

Bold-faced values correspond to the approximate Pareto front and values in brackets correspond to the quasi-reference Pareto front.

Table 2
Scaled objective values corresponding to the best solution found in each of the ten algorithmic executions.

<table>
<thead>
<tr>
<th>Number</th>
<th>99Mo</th>
<th>Silicon</th>
<th>IPR-1</th>
<th>IPR-2</th>
<th>z_k(X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.3%</td>
<td>9.0%</td>
<td>5.6%</td>
<td>16.7%</td>
<td>0.15852</td>
</tr>
<tr>
<td>2</td>
<td>6.0%</td>
<td>8.4%</td>
<td>6.6%</td>
<td>17.6%</td>
<td>0.15376</td>
</tr>
<tr>
<td>3</td>
<td>6.0%</td>
<td>8.4%</td>
<td>6.6%</td>
<td>17.6%</td>
<td>0.15394</td>
</tr>
<tr>
<td>4</td>
<td>6.0%</td>
<td>8.4%</td>
<td>6.6%</td>
<td>17.6%</td>
<td>0.15361</td>
</tr>
<tr>
<td>5</td>
<td>6.3%</td>
<td>8.7%</td>
<td>5.9%</td>
<td>17.4%</td>
<td>0.15961</td>
</tr>
<tr>
<td>6</td>
<td>6.3%</td>
<td>8.2%</td>
<td>5.8%</td>
<td>16.8%</td>
<td>0.15759</td>
</tr>
<tr>
<td>7</td>
<td>6.3%</td>
<td>10.2%</td>
<td>6.0%</td>
<td>17.0%</td>
<td>0.15527</td>
</tr>
<tr>
<td>8</td>
<td>6.0%</td>
<td>8.7%</td>
<td>7.1%</td>
<td>16.9%</td>
<td>0.15162</td>
</tr>
<tr>
<td>9</td>
<td>6.3%</td>
<td>9.3%</td>
<td>7.1%</td>
<td>17.8%</td>
<td>0.14646</td>
</tr>
<tr>
<td>10</td>
<td>6.3%</td>
<td>8.3%</td>
<td>5.3%</td>
<td>16.0%</td>
<td>0.16007</td>
</tr>
</tbody>
</table>
IPR-2 objective, these results show that moderate improvements in the IPR objectives are almost always possible within the approximate Pareto set.

Unfortunately, the best solution found in terms of the $^{99}$Mo objective yields a performance decrease of 2.7% from the historical solution. The corresponding range of available values lies between −6.3% and −2.7%. Thus, any solution within the approximate Pareto set sacrifices performance in the $^{99}$Mo objective with respect to the historical solution. The historical solution is, however, largely geared towards maximisation of $^{99}$Mo. Therefore, newly imposed constraints and/or the optimisation of other objectives simultaneously would likely always deteriorate the $^{99}$Mo objective. Fortunately in this problem instance, the range of the $^{99}$Mo objective values is smaller when compared to the range of improvements within the other objectives. Furthermore, it is observed that the performance improvements suggested by the proposed methodology are comparable to those available in the quasi-reference Pareto set. As such, the methodology was again able to suggest a good set of trade-off solutions within a limited computational budget in response to a change of utilisation requirements for SAFARI-1.

In addition to the payoff table, the scaled objective function values corresponding to the best solution found in each of the ten algorithmic executions are presented in Table 2. It is observed that the best solutions found in each case achieve very similar values for the objectives, namely improvements of approximately 8%, 6% and 17% in the silicon, IPR-1 and IPR-2 objectives, respectively, at the cost of decreasing the $^{99}$Mo objective by approximately 6%.

The overall best performance (with respect to $z_q$) in Table 2 has been shaded in grey. In Fig. 7, the reload configurations of the historical solution and this overall best solution for Problem 3 are presented visually in terms of the uranium-235 mass in each fuel assembly. It may be observed that the best-found configuration assigns heavier-massed assemblies near the IPR positions (D6 and F6) in the core, as opposed to the historical solution. The historical solution is, however, largely geared towards maximisation of $^{99}$Mo. Therefore, newly imposed constraints and/or the optimisation of other objectives simultaneously would likely always deteriorate the $^{99}$Mo objective. Fortunately in this problem instance, the range of the $^{99}$Mo objective values is smaller when compared to the range of improvements within the other objectives. Furthermore, it is observed that the performance improvements suggested by the proposed methodology are comparable to those available in the quasi-reference Pareto set. As such, the methodology was again able to suggest a good set of trade-off solutions within a limited computational budget in response to a change of utilisation requirements for SAFARI-1.

An augmented Chebyshev scalarising objective function using aspiration levels is employed in the methodology for suitably modelling multiple objectives. It does so by addressing the shortcomings of the popular weighting method for an appropriate choice of aspiration levels. Furthermore, an additive penalty function as constraint handling technique enables the methodology to cater for a variety of constraints. In order to add more value to the optimisation result found by minimising the scalarising function, an archive of nondominated decision vectors is maintained during optimisation. In this way, nondominated decision vectors in the vicinity of the final approximate solution (possibly Pareto optimal) may be uncovered and presented to decision makers as an approximate Pareto front of trade-off solutions. This allows for much more flexibility in decision making.

A metaheuristic technique, called harmony search, has been adapted for ICFMO and may be used to solve the ICFMO problem within the methodology proposed here. This includes the capability of yielding a nondominated set of solutions as result. The adapted HS algorithm may also serve as an alternative to the HS algorithms currently employed in the ICFMO literature. The results presented in this paper demonstrate that the algorithm is relatively robust with respect to optimising different ICFMO problems and under different starting conditions.

The unified methodology may be especially useful for research reactors responding to changing safety, research and commercial demands with relatively short turnaround times. As such, the methodology was applied to three ICFMO test problems of varying nature for the SAFARI-1 research reactor in order to demonstrate its applicability. It was found that the single- and multiobjective test problems can be adequately modelled within the methodology.

Furthermore, the solution technique adopted performs well within a limited computational budget of 1000 objective function evaluations. For the SOP test problem, a significant improvement of 29.5% in the excess reactivity of the reactor was obtained. An approximate Pareto set was obtained for the two-objective test problem in which improvements of up to 6% in the performance of beam tubes $1 + 2$ were possible, at the cost of decreases of up to 12% in the performance of beam tube 5. Likewise, an approximate Pareto set was also obtained for the four-objective test problem. In that case, improvements of up to approximately 18%, 13% and 20% for three of the respective objectives were obtained, at the cost of decreases up to 6% in the remaining objective. The results for all three test problems were also comparable in accuracy to quasi-reference solutions obtained according to the procedures described earlier.

It is therefore concluded that the proposed unified methodology for ICFMO may be used as an effective decision support tool for reactor operators tasked with designing reload configurations from cycle to cycle. Since a general modelling approach has been followed, it is expected that the proposed methodology would also

7. Conclusions

In this paper, a unified methodology has been proposed for the modelling and solution of single- and multiobjective ICFMO problems, catering for both constrained and unconstrained problem instances. The methodology is easy to use due to the small number of user-defined input parameters that are required, along with the fact that only one model is necessary for all variants of the ICFMO problem, and one solution technique solves it. With this methodology, ICFMO problems incorporating a variety of objectives and/or constraints may be modelled and solved rapidly, thus providing cycle-to-cycle optimisation capabilities for nuclear reactors.
prove effective for research reactors other than SAFARI-1, as well as for power reactors.

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